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NAVAL POSTGRADUATE SCHOOL

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THESIS

AN INVESTIGATION OF THE
COMBAT AIR PATROL STATIONING
IN AN INTEGRATED AIR DEFENSE SCENARIO

by

Ernani Brisolla Jordao

December 1990

Thesis Advisor:

Donald P. Gaver

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91-13947



91 10 24 090

REPORT DOCUMENTATION PAGE				
1a REPORT SECURITY CLASSIFICATION Unclassified		1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b DECLASSIFICATION/DOWNGRADING SCHEDULE				
4 PERFORMING ORGANIZATION REPORT NUMBER(S)		5 MONITORING ORGANIZATION REPORT NUMBER(S)		
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School	6b OFFICE SYMBOL (If applicable) 30	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School		
6c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000		7b ADDRESS (City, State, and ZIP Code) Monterey, CA 93943 5000		
8a NAME OF FUNDING/SPONSORING ORGANIZATION	8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c ADDRESS (City, State, and ZIP Code)		10 SOURCE OF FUNDING NUMBERS		
		Program Element No	Project No	Task No Work Unit Accession Number
11. TITLE (Include Security Classification) An Investigation of the Combat Air Patrol Stationing in an Integrated Air Defense Scenario (unclassified)				
12. PERSONAL AUTHOR(S) Jordao, Ernani B.				
13a. TYPE OF REPORT Master's Thesis	13b. TIME COVERED From To	14. DATE OF REPORT (year, month, day) December 1990	15. PAGE COUNT 128	
16. SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
17. COSATI CODES		18. SUBJECT TERMS (continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUBGROUP		
		Combat Air Patrol, Air Defense, Air Intercept, Air-to-Air Combat, Differential Equations, Combat Models, Deterministic Model, Integrated Air Defense		
19. ABSTRACT (continue on reverse if necessary and identify by block number)				
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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT		21. ABSTRACT SECURITY CLASSIFICATION		
<input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS REPORT <input type="checkbox"/> DTIC USERS				
22a. NAME OF RESPONSIBLE INDIVIDUAL Donald P. Gaver		22b. TELEPHONE (Include Area code) (408) 646-2605		22c. OFFICE SYMBOL OR/Gv

Approved for public release; distribution is unlimited.



An Investigation of the Combat Air Patrol Stationing
in an Integrated Air Defense Scenario

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Submitted in partial fulfillment
of the requirements for the degree of

Accession For	
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DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
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Distribution/	
Availability Codes	
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MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

December 1990

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ABSTRACT

This investigation of the Combat Air Patrol stationing problem analyzes the geometry of a hypothetical tactical scenario. Expressions to determine the following quantities are derived: (1) the minimum early warning radar detection range required for intercept feasibility; (2) intercept range as measured from the target area position when the interception begins from the CAP station; and (3) the minimum number of interceptors required to actively maintain one CAP station; The time variables most relevant to the problem are identified and investigated in the context of interceptor fuel consumption.

The complexities of the dynamic process embedded in a Combat Air Patrol management are modelled by means of a deterministic macro model. The state variables portraying both the logistic and the operational aspects involving the CAP activity are defined; system parameters controlling the transition flow from one state to another are presented to represent the constraints of realities. A method for computing the attrition rate based on Bonder and Farrell's methodology is derived. Numerical examples are presented and the results analyzed. The application of such a CAP stationing analysis model for air defense planning is discussed.

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I. INTRODUCTION

Effective planning decisions for best use of sparse defensive assets in a forward air defense scenario require both proper analysis and thorough understanding of how attack and defense interact. These interactions occur in many different ways, most of them requiring a specific analysis appropriate to the context. Such analysis will offer improved understanding of the capabilities, and specific weaknesses of the resources available to accomplish a given mission. It will thereby contribute to realistic planning and should lead to appropriate tactical decisions.

This thesis is a study on the Combat Air Patrol (CAP) location problem in a Forward Air Defense (FAD) scenario.

A. SCENARIO

A set of sensitive points located in a specified geographical location called *target area* must be defended against air attacks. The air attacks are of the penetration--strike type of attack performed by fighter-bomber aircraft, which arrive at the target area through a known sector of penetration. There is an integrated air defense system (IADS) to protect the target area from these air attacks. This system comprises a set of ground-based anti-air weapons located in the target area, an integrated air defense network that provides early warning radar (EWR) detection and C³ capabilities, and a given number of air interceptor fighters deployed in an

air base with specified location. Each air interceptor fighter departs from the air base and is kept in Combat Air Patrol (CAP) in a position called CAP station located forward into the sector of penetration. From the CAP station, the air interceptor fighter is either engaged in the interception of an arriving raid, or he returns to the air base if no raid appears during the period of time he can stay on CAP station. Around the target area there is a volume of air space determined by the effective range of the anti-air weapons defending the area. The air interceptor fighters must not fly through this air space volume to avoid being shot at by friendly weapons. To accomplish their mission the fighters must destroy or neutralize as many as possible of the attacker aircraft at or before the perimeter of this volume.

B. PROBLEM DEFINITION

Considering the scenario described above, the problem to be addressed in this study is described as follows:

Given:

the location of the target area;

the location of the air base;

the angular sector of raid penetration measured with center at the target area location;

the early warning range of the radar net measured from the target area location;

the effective range of the anti-air weapons defending the target area; and,

the maximum number of air interceptor fighters available in the defense inventory,

determine the location to place the CAP station so as to maximize the expected number of raids destroyed/neutralized by the air interceptor fighters, before the raids reach the anti-air defense line.

C. OBJECTIVES

The objectives of this study are:

1. to analyze the problem as stated above and identify the key factors affecting the selection of CAP station location;
2. to derive a deterministic model by means of a system of differential equations representing the scenario in scope;
3. to use the deterministic model to assess how the identified key factors impact the effectiveness of the air interceptor fighters at different CAP station locations.

II. BACKGROUND

A. PROBLEM CHARACTERISTICS

Air interceptor fighters are used in the air defense mission in two distinct ways: as Ground Alert Interceptors (GAI) or in forward Combat Air Patrols (CAP). Shaw [Ref. 1] gives a description of the aspects to be considered for each of these options.

According to Shaw the selection of which mode to use depends on the situation at hand. Factors to be considered in making this decision are:

- (1) type of raid expected;
- (2) number of targets to protect;
- (3) degree of certainty about the attacker's approaching route;
- (4) early warning distance;
- (5) characteristics, performance and availability of the air interceptor fighters, and;
- (6) threat characteristics such as attacker's air speed and weapons release range.

One reason for using CAP is to achieve raid interception at an advantageous distance from the target area, providing more time to destroy or neutralize it before the raid reaches its objectives. In general, the employment of CAP is more expensive than ground-based intercept because of the fuel consumption and crew requirements to maintain airborne defensive posture for prolonged periods between attacks; also, it makes the air defense problem more complex as it demands extra capability from the C³ system. The use of CAP may be inefficient and ineffective if it is not

appropriately deployed and managed, yet it can be very effective under the right operating parameters. It is the purpose of the analysis to identify those operating parameters for a proper tactical disposition. Sometimes establishment of CAP is the only viable alternative for the decision maker, as would be the case if the early warning distance of an attack is expected to be insufficient to launch a GAI and intercept the raid at useful range from the target. Again, analysis should be performed to decide upon a wise CAP disposition.

Once the decision for CAP employment is made one contemplates the following issues:

- (1) the distance from target to CAP station,
- (2) the CAP altitude;
- (3) patrol technique; and
- (4) command-and-control-specific procedures.

The practical distance from target to CAP depends on factors such as the number of aircraft available, the area that must be covered, and the useful time on station for the patrolling aircraft. These factors are affected, respectively, by the logistics of the Air Intercept Squadron, by the performance of the interceptors' on-board sensors, and by the endurance of the interceptors and air-refuelling possibilities .

The choice of CAP altitude must consider the expected altitude of the threat, the interceptor's weapons system characteristics, and environmental conditions. This

choice must be made so as to optimize the chances of the intruder detection, and the thwarting of his attack.

The considerations involved in selecting the patrol technique for the CAP are: endurance; optimization of sensor and visual coverage; weapons capabilities; and defense against attack by enemy fighter sweep or fighter escort. Usually the defense faces a shortage of aircraft to maintain what it considers an adequate number of aircraft actually on CAP station. Nevertheless, a minimum number of interceptors per CAP station should be considered. The number of aircraft per patrol is dictated by situation assessment; two aircraft per CAP station is usually considered the minimal force level on station.

The command and control procedures required to make a CAP effective may be very complex and demanding. The CAP demands from the defense C³ network the ability to perform long-range jam resistant target detection and identification, long-range communications with the interceptors on CAP station as well as long-range intercept control capability. Considering the fact that the CAP is not the only activity being controlled at a time by the C³ system, both pilots and controllers must be aware of any special radio-communication procedure for the CAP, as well of the rules of engagement and type of control for intercept.[Ref. 1:pp. 325-330]

B. LITERATURE REVIEW

Some air-to-air models found in the unclassified literature will now be presented. For simplicity, those models more germane to the nature of the problem investigated here are described in this section. The description of others air-to-air models can be found in Appendix A.

1. Air-to-Air Battle Models

A comprehensive study of the interactions between defense and offense in the air-to-air battle can be found in Heilenday [Ref. 2]. Many aspects of an air defense system and air vehicle penetration are analyzed. Therein the author begins by addressing the basic concepts of the offense and defense missions, and discussing radar and electro-optics (EO) fundamentals. Then the offensive/defensive interactions are analyzed and the basic defense actions are identified as: initial target assignment; airborne interceptor actions; and SAM/AAA intercept. The fundamental elements regarding the air interceptor actions are identified and represented as time measures and probabilities. A model for the probability of an air interceptor (AI) to kill a penetrator with a single shot (PK) is given as a combination of conditional probabilities, as follows:

$$PK = P_{ai} P_v P_d P_t P_c P_i SSPK$$

where:

P_{ai} = Probability AI available and alerted;

P_v = Probability AI is correctly vectored, given that it has been alerted;

P_d = Probability AI detects, given correct vectoring;

P_t = Probability AI properly tracks, given detection;

P_c = Probability AI converts, given track;

P_l = Probability AI launches weapon, given conversion;

SSPK = Single Shot Kill Probability for one shot [Ref. 2:p. 9-6]

The ways an interceptor may attack a penetrator are described according three different attack patterns:

Radar Head-on - a head-on approach using the AI radar as the sensor and an attack with a salvo of two radar guided missiles;

Radar Tail-on - a tail-on approach using the AI radar as the sensor and an attack with a salvo of two radar guided missiles;

Infra-red (IR)/Visual - a tail-on approach using EO sensors (IR/visual) and an attack with a salvo of two IR missiles [Ref. 2:p. 9-11].

Based on these attack patterns, attrition models are derived. First a one-on-one P_k is presented for each initial attack pattern attempt, and considering a initial head-on attack and a initial tail-on attack. For each of these initial attacks, subsequent reattacks are considered depending on fuel and ammunition availability on the AI.

The models are as following:

$$P_{k_{T/E}} = P_{d_{T/E}} \{1 - P_{S_{T/IR}} P_{S_{T/gun}}\}$$

$$P_{k_{H/E}} = P_{d_{H/E}} \{1 - P_{S_{H/IR}} P_{S_{H/gun}} P_{S_{T/IR}} P_{S_{T/gun}}\}$$

$$P_{k_{T/R}} = P_{d_{T/R}} \{1 - P_{S_{T/R}} P_{S_{T/IR}} P_{S_{T/gun}}\}$$

$$P_{k_{H/R}} = P_{d_{H/R}} \{1 - P_{S_{H/R}} P_{S_{T/R}} P_{S_{T/IR}} P_{S_{T/gun}}\}$$

where:

$P_{k_{T/E}}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Tail-on Electro-optical detection;

$Pk_{H/E}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Head-on Electro-optical detection;

$Pk_{T/R}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Tail-on Radar detection;

$Pk_{H/R}$ = one-on-one probability of penetrator kill by a single AI beginning with an attempted Head-on Radar detection.

In these models the entire sequence of AI weapons attack is considered as dependent upon the initial detection (and track) probability, represented by the Pd terms in each equation. The PS terms represent, each, the probability of penetrator survival after each AI weapon attack/pass, what is considered as independent of the success of the previous pass.[Ref. 2:pp.16-2 - 16-5] The author uses these models to evaluate the results of each attack pattern under undegraded and degraded conditions. Six degradation categories are considered, according to the possible penetrator tactics:

- (1) Electronic countermeasures (ECM)
- (2) Infra-red countermeasures (IRCM)
- (3) Optical camouflage
- (4) Evasive maneuvers
- (5) Low radar cross section
- (6) Lethal self defense

Heilenday also analyzes the scenario of many penetrators versus many AI's. To this end, the number of AI assignments required to service penetrators is assessed with and without considerations to defense resources and capabilities. The problem of multiple air interceptor types is addressed, as well as the issue of multiple types of

penetrators and the preferential assignments against certain penetrators[Ref. 2:pp.17-1 - 17-20]. In summary, this is a comprehensive study of the air-to-air battle, with detailed analysis of the defense/offense interactions, but the CAP station location problem is not addressed by Heilenday.

Grant [Ref. 3] investigated the effects of command and control on the Forward Air Defense (FAD). The study develops a basic methodology for modeling the effects of command and control on the FAD. It is modeled from the Soviet perspective to judge the effectiveness of the defense against a US penetrating force. In her study a review of some FAD and bomber penetration models is presented. The main characteristics of one of these models is presented here. A general description of the other models studied by Grant can be found in the Appendix.

Corridor Penetration Model (COPEM) This model was developed at Stanford Research Institute (SRI) as part of a study to improve the representation of airborne strategic systems in aggregated effectiveness evaluation models. It is a sophisticated analytic model divided into two sections: the forward air defense model and the weapon/target allocation model. The model finds the probability a penetrator reaches a certain depth in the forward air defense zone before being destroyed. The zone is divided into a rectangular grid of cells, interceptors are distributed across the grid according to some probability distribution, and penetrators enter and fly through the grid in straight lines parallel to the sides. The number of intercepts which can be made depends on where the penetrator is detected and how many intercept attempts the control center can then make in the time remaining with the

interceptors available before the penetrator exits the grid. According to the author, the model makes the assumption that this process can be represented as non-homogeneous Poisson process with a time dependent parameter and, for this assumption to hold, in some cases the interceptor must be loaded with an unrealistic number of weapons. For a detailed description of the forward air defense part of COPEM and a discussion about the validity of the underlying assumptions of the model, see Grant [Ref. 3:pp. 111-132].

In what follows we have the main aspects of a comprehensive investigation focussed on the probability of an aircraft being killed in a hostile environment in term of aircraft survivability. This work is presented by Ball [Ref. 4]. According to him, the probability of kill of the aircraft is the product of the susceptibility and the vulnerability, or

$$\text{Probability of Kill} = \text{Susceptibility} * \text{Vulnerability} \text{ [Ref. 4:p. 2]}$$

In this context, susceptibility is defined as the probability of the aircraft being hit, P_H , or as "the inability of an aircraft to avoid being damaged in the pursuit of its mission" [Ref. 4:p. 223]. Vulnerability "refers to the inability of the aircraft to withstand one or more hits by damage mechanism, to its vincibility, to its liability to serious damage or destruction when hit by enemy fire" [Ref. 4:p. 135]. Ball models vulnerability as the probability of kill given a hit, $P_{K/H}$. This way the Probability of Kill, P_K is written as

$$P_K = P_H P_{K/H}.$$

The probability of the aircraft being hit, P_H , is the product of individual probabilities, some of which are conditional on the result of a previous event:

$$P_H = P_A P_{DIT} P_{LGD} ,$$

where:

P_A = probability that the threat is active and ready to engage the aircraft;

P_{DIT} = probability that the aircraft is detected, identified, and tracked by the threat given the threat is active;

P_{LGD} = probability that a threat propagator is launched or fired, possibly guided, and either hits the aircraft or a high-explosive warhead is detonated sufficiently close to the aircraft to cause a hit by a damage mechanism.[Ref. 4:p. 1]

The author gives a detailed discussion with respective model derivation for each of the above probabilities for a comprehensive threat spectrum [Ref. 4:pp. 223-306]. Further, on the assumption that the aircraft has been detected and that a threat propagator has been launched or fired, a model for the probability of aircraft kill given a single shot, P_{KSS} is presented as

$$P_{KSS} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y) P_f(x, y) V(x, y) dx dy$$

where:

$\rho(x, y)$ = miss distance frequency distribution;

$P_f(x, y)$ = probability of fusing for an HE (high explosive) warhead;

$V(x, y)$ = kill function that defines the probability the target is killed due to a propagator whose trajectory intersects the intercept plane at x, y .

Equations for the P_{KSS} of different types of warheads are discussed and presented.[Ref. 4:pp. 315-319]. Next, Ball addresses the issue of one-on-one

survivability, i.e., the probability that an aircraft survives an encounter with a single threat, $P_{S/E}$, which is modeled as

$$P_{S/E} = 1 - P_{K/E} = 1 - \bar{P}_D P_L P_{KSS}$$

where:

\bar{P}_D = measure of detection, i.e., the probability that the aircraft has been detected (at least once) from the start of a search up to the present time t ;

P_L = probability that a propagator will be launched or fired at the aircraft;

P_{KSS} = probability of kill given a single shot, as defined above;

$P_{K/E}$ = probability that the aircraft is killed in an encounter in which one propagator may be fired or launched at some time t .

Based on the previous model, the probability the aircraft survives the N shot encounter is then derived:

$$P_{S/E} = 1 - \bar{P}_D P_L \left[1 - \prod_{i=1}^N (1 - P_{KSS_i}) \right]$$

where i denotes the i^{th} shot. [Ref. 4:pp. 319-321]. After the encounter survivability, the sortie survivability is discussed. A model is derived for the probability the aircraft survives the E encounters on the sortie, $P_{S/S}$, as following

$$P_{S/S} = \prod_{i=1}^E P_{S/E_i} = \prod_{i=1}^E (1 - P_{K/E_i})$$

where:

P_{S/E_i} = survival probability for each encounter;

P_{K/E_i} = kill probability of each encounter;

E = sum of those encounters that occur as the aircraft flies through any zone defenses to get to the target, those that occur near any point defended targets, and those that occur as the aircraft returns through the same defended zone.

If more than one type of weapon is encountered during the sortie, the model for the sortie survival probability becomes

$$P_{S/S} = \left[\prod_{i=1}^{E_1} (1 - P_{K/E_i})_1 \right] \left[\prod_{i=1}^{E_2} (1 - P_{K/E_i})_2 \right] \dots \left[\prod_{i=1}^{E_m} (1 - P_{K/E_i})_m \right]$$

where:

E_1, E_2, \dots, E_m = number of independent encounters with weapon types 1, 2, ... m , respectively, and

$(1 - P_{K/E_i})_j$ = probability of survival of the i^{th} encounter with the j^{th} weapon type.
[Ref. 4:pp. 321-323]

2. CAP Stationing Model

A formulation for the disposition of CAP station problem is found in Naval Operations Analysis [Ref. 5]. The scenario assumed is one of a naval task force in a mid-ocean location, where the aircraft carrier, at the center of the task force, is the main target to protect. From the aircraft carrier the interceptors are launched and kept on CAP stations from which they are engaged in the interception of penetrators flying toward the center of the task force. Considering the center of the task force as the origin of a cylindrical coordinate system the CAP stations are located equidistantly on a circle of radius d from the task force center. There is one assumption that drives the whole formulation of the problem, namely that the probability of penetrator kill is a non-decreasing function of the range of interception

as measured from the center of the task force. This means that the further away from the protected target the interception takes place, the higher the probability of killing the penetrator. The distribution of the probability of killing the penetrator given the interception range is assumed to be a user-supplied function. Another underlying fact upon which the formulation is based is that the radar horizontal first-detection range is a random variable whose density function is known. Based on these facts, an expression is derived for the interception range as a function of both the first horizontal detection range and the CAP station location relative to the task force center. In the derivation of this expression the interceptor is assumed to be flying at the same air speed as the penetrator. Having the interception range thus expressed and assuming that the raids are equally likely to approach from any direction, the expected probability of kill is then derived as a function of the number of CAP stations and of the distance of CAP station from the task force center.[Ref. 5:pp. 220-223] The strength of this formulation is the fact that it express the probability of penetrator kill as a function of CAP station distance from the target to be protected as well as a function of the number of CAP stations used, which is likely to be a useful tool for planning purposes. On the other hand, by assuming, in the derivation, that interceptor and penetrator fly both at the same air speed, the solution loses generality. Other aspects not considered in this model are the endurance, and the maximum intercept range, of the interceptor. In the next chapter an expression for the interception range will be derived where different air speeds for interceptor and penetrator are considered.

III. CAP STATIONING ANALYSIS

The scenario described in Chapter I will be used as framework for developing a model which permits investigation of the CAP stationing problem.

The first issue to address in this problem is to be able to identify the situations in which the use of the interceptors in a CAP disposition is actually required. Once the necessity of CAP is verified the next question to answer is how many CAP stations could be permanently activated, given the number of interceptors available. Before these two issues are addressed we will define the variables to be used in the formulations.

A. VARIABLES DEFINITION

Let us define the following variables:

v_a = air speed of the attacker aircraft;

v_i = air speed of the interceptor aircraft;

t_{ID} = time elapsed from the moment the attacker is first detected by the radar until it is positively identified as a hostile;

t_{AI} = time elapsed from the moment an interceptor in GAI posture is scrambled until it takes off from the air base;

t_{intc} = time elapsed from the moment an interception begins until the moment the interceptor engages in air-to-air combat with the attacker;

t_{cmb} = maximum time period for which the interceptor can stay engaged in air-to-air combat with the attacker;

t_{rec} = length of time period required by the interceptor to fly back to the air base from the point it finishes its mission;

t_{oc} = time length it takes to the interceptor to fly from the air base out to the CAP station;

t_{bc} = time length it takes for the interceptor to fly from the CAP station back to the air base;

t_{be} = time length it takes for the interceptor to fly from the point it disengages air-to-air combat with the attacker back to the air base;

c_R = number of repair crews available in the air base;

t_{rep} = time length it takes for one repair crew to repair one aircraft;

t_m = maximum length of time a interceptor can stay on station if no attacker arrives.

t_a = average time interval between two consecutive attacker's arrivals.

Now consider Figure 1. In this Figure consider the target location, T, as the origin of a cylindrical coordinate system. Let C be the air base location. The following quantities are distance measures, taken from the origin, T, of this coordinate system:

R = radar horizontal first detection range, i.e., distance from the target at which the attacker is first detected by the C³ network;

I = identification range, i.e., distance from the target at which the attacker is identified as hostile;

P = engagement range, i.e., distance from the target the attacker is when the interception begins;

d = distance from the target location to the air base location;

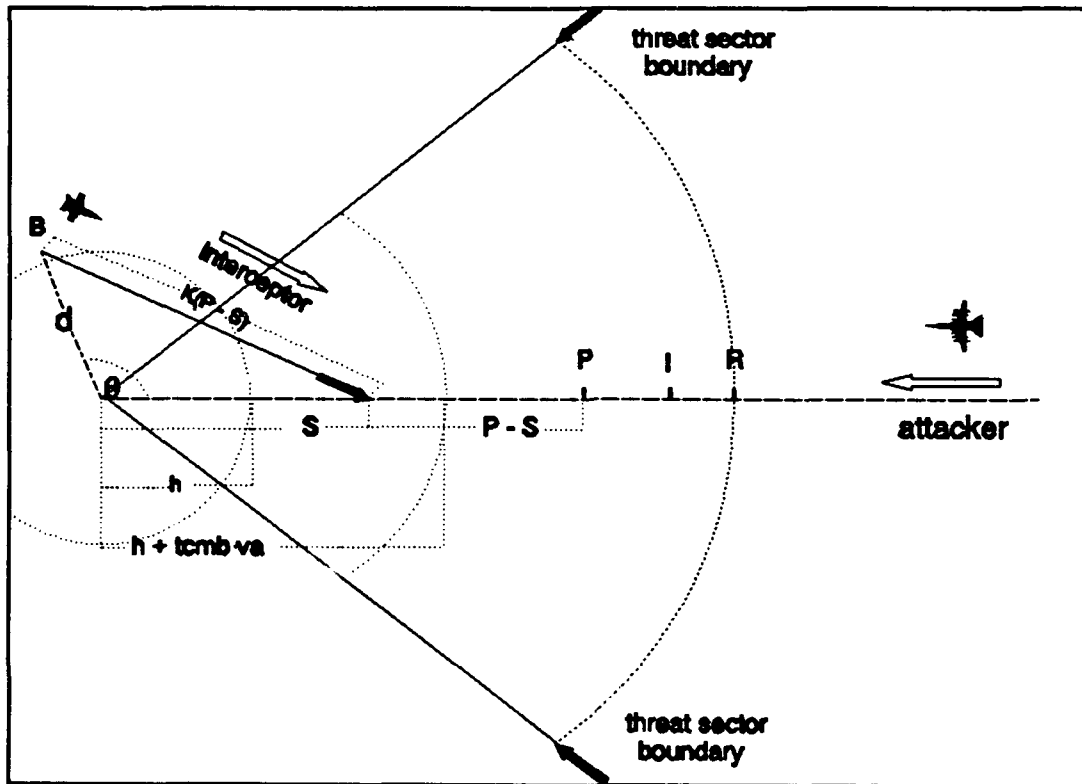


Figure 1 Ground Alert Interceptor (GAI) Scenario

h = anti-air weapons effective range, i.e., minimum distance from the target at which the attacker must have been destroyed/neutralized by the interceptor.

Let θ be the angle formed by the attacker's flight path through the target location and the line segment connecting the target location and the air base location; $\theta \geq 0$. Define

t_D = delay time measured from the moment of attacker's first radar detection until the moment the interception begins.

$$t_D = t_{ID} + t_{AI}.$$

Let

$$K = \frac{v_i}{v_a}, K > 0$$

K represents the air speed relationship between interceptor and attacker.

B. IDENTIFYING CAP REQUIREMENT

To identify the conditions requiring the use of interceptors in a CAP disposition we will use the same analytical methodology as in Naval Operations Analysis [Ref. 5:p. 221]. The differences from that scenario to the one used here are: the interceptor's air base is not located in the target area; the air speed of the attacker aircraft is not necessarily equal to the interceptor's air speed.

In Figure 1 consider the air base located at B with the interceptors in GAI and a reaction time of t_{AI} units of time. Further assume that an attacker is first detected at range R from the target T. It will take t_{ID} units of time for the C^3 system identify the attacker as a hostile and scramble the interceptor. When the interceptor is scrambled the attacker will be at range I from the target, and it will take t_{AI} units of time for the interceptor to take off. Assuming that the interception procedure starts immediately after take off, this means that the attacker will be at range P from the target at the beginning of the interception. If the interception occurs at range S from the target and considering the air speed relationship, K, between interceptor and attacker, we can see that during the interception the attacker will travel a distance

equals to (P-S) while the interceptor travels a distance equals to K(P-S). Applying the law of cosines to the triangle we have:

$$K^2(P - S)^2 = d^2 + S^2 - 2dS\cos\theta$$

$$K^2(P^2 - 2PS + S^2) = d^2 + S^2 - 2dS\cos\theta$$

$$K^2P^2 - 2K^2PS + K^2S^2 = d^2 + S^2 - 2dS\cos\theta$$

$$K^2S^2 - S^2 + 2dS\cos\theta - 2K^2PS = d^2 - K^2P^2$$

$$S^2(K^2 - 1) + 2S(d\cos\theta - K^2P) = d^2 - K^2P^2$$

$$S^2 + 2S \left[\frac{d\cos\theta - K^2P}{K^2 - 1} \right] = \frac{d^2 - K^2P^2}{K^2 - 1}$$

$$S^2 + 2S \left[\frac{d\cos\theta - K^2P}{K^2 - 1} \right] + \left[\frac{d\cos\theta - K^2P}{K^2 - 1} \right]^2 = \frac{d^2 - K^2P^2}{K^2 - 1} + \left[\frac{d\cos\theta - K^2P}{K^2 - 1} \right]^2$$

$$\left[S + \left[\frac{d\cos\theta - K^2P}{K^2 - 1} \right] \right]^2 = \frac{(K^2 - 1)(d^2 - K^2P^2) + d^2\cos^2\theta - 2d\cos\theta K^2P + K^4P^2}{(K^2 - 1)^2}$$

after some algebraic manipulations, and solving the above equation for S, we get

$$S = \frac{K^2P - d\cos\theta \pm \sqrt{K^2P^2 - 2d\cos\theta K^2P + d^2\cos^2\theta + K^2d^2 - d^2}}{K^2 - 1} \quad (3.1)$$

Equation (3.1) expresses S as a function of K, P, d and θ . The first thing to note concerning this equation is the double possible solution. Second, we observe that, with respect to K, S must be continuous for all $k > 0$. Hence to find the value of S when $K = 1$ we have to find the value of S in the limit, as $K \rightarrow 1$. Evaluating these limits we find:

$$\lim_{K \rightarrow 1} S_1 = \frac{P - d \cos \theta + \sqrt{(P - d \cos \theta)^2}}{1 - 1} = \frac{2P - 2d \cos \theta}{0}$$

$$\lim_{K \rightarrow 1} S_2 = \frac{P - d \cos \theta - \sqrt{(P - d \cos \theta)^2}}{1 - 1} = \frac{0}{0}$$

When $K=1$ the solution S_1 is not defined and the solution S_2 takes an indeterminate form. Because S has to be continuous for all $K > 0$, S_1 can not be a solution for the value of S . By applying l'Hôpital's rule on S_2 we obtain the value of S when $K=1$. Hence we have the following expressions for the values of S

$$S = \frac{K^2 P - d \cos \theta - \sqrt{K^2 P^2 - 2d \cos \theta K^2 P + d^2 \cos^2 \theta + K^2 d^2 - d^2}}{K^2 - 1},$$

if $(K > 0) \wedge (K \neq 1)$, and

$$S = \frac{P^2 - d^2}{2(P - d \cos \theta)}, \text{ if } (K = 1) \wedge (P \neq d \cos \theta).$$

It can be verified that the above expression for S when $K = 1$ is consistent with the derivations in Naval Operations Analysis¹ [Ref. 5:p. 222].

Now, to obtain an expression for S as a function of the first detection range, R , we use the fact that $F = R - t_D \cdot v_a$. Substituting this expression for P into the two previous equations, we get:

¹ See equation 11-5, p. 222 in that publication.

$$S = \frac{K^2(R - t_D v_a) - d \cos \theta}{K^2 - 1} - \frac{\sqrt{K^2(R - t_D v_a)^2 - 2d \cos \theta K^2(R - t_D v_a) + d^2 \cos^2 \theta + K^2 d^2 - d^2}}{K^2 - 1}, \quad (3.2)$$

when $(K > 0) \wedge (K \neq 1)$, and

$$S = \frac{(R - t_D v_a)^2 - d^2}{2[(R - t_D v_a) - d \cos \theta]}, \quad \text{when } (K = 1) \wedge (R \neq d \cos \theta + t_D v_a). \quad (3.3)$$

Note that when $(K > 0) \wedge (K \neq 1)$, S is defined if and only if the expression under the radical is non-negative. This imposes a third condition on the values of K , namely

$$K \geq \left| \frac{d \sin \theta}{\sqrt{(R - t_D v_a - d \cos \theta)^2 + d^2 \sin^2 \theta}} \right|, \quad (3.4)$$

From the air defense view point, an interception is defined as a *valid intercept* if and only if the interceptor is able to fire his weapons at the attacker at or before the point where the interceptor's presence would restrict the use of other defense weapons. This implies that the interceptor will have accomplished his mission if and only if the attacker is destroyed/neutralized at or before a distance h from the target. For this to happen, the interception must be completed at a distance from the target such that there is enough time for the interceptor to engage the air-to-air combat with the attacker and employ its weapons. Thus we can see from Figure 1 that the minimum value of S that permits the attacker destruction/neutralization at or before the range h from the target occurs when $S = h + t_{cmb} \cdot v_a$ (the rationale for this

expression for S will be addressed later in this study). This fact allows us to derive an expression for the minimum value of the radar first detection range, R , for which a valid interception is possible. Using again the law of cosines in the triangle of Figure 1, we have

$$K^2(P - S)^2 = d^2 + S^2 - d \cos \theta$$

and solving this equation for P we obtain

$$P = S \pm \frac{\sqrt{(d - S \cos \theta)^2 + S^2 \sin^2 \theta}}{K} \quad (3.5)$$

To interpret the double solution of equation (3.5) we refer to Figure 1 and consider the following fact. In a given moment an attacker may be flying either inbound (toward the target) or outbound (away from the target). Depending on the values of K and θ , it is possible for the interceptor to attempt a tail-on interception and catch up with the attacker when it is flying outbound. But see that, in such a case, we will have $S > P \forall \theta$ for which the interception is possible. On the other hand, if the attacker is flying inbound, we will have $S \leq P \forall \theta$ for which the interception is possible. In this study we are interested only on those cases in which the attacker is flying inbound. Hence we conclude that the solution of interest for equation (3.5) is

$$P = S + \frac{\sqrt{(d - S \cos \theta)^2 + S^2 \sin^2 \theta}}{K} \quad (3.6)$$

Let

R_{\min} = the minimum value of the radar first detection range for which a valid intercept is possible.

Now, consider the two facts:

(1) the minimum intercept range for a valid interception occurs when

$$S = h + t_{\text{cmb}} \cdot v_a; \text{ and}$$

$$(2) P = R - t_D \cdot v_a.$$

Substituting these expressions into equation (3.6) and arranging the terms, we obtain

$$R_{\min} = h + v_a(t_D + t_{\text{cmb}}) + \frac{\sqrt{[d - (h + t_{\text{cmb}} v_a)]^2 + (h + t_{\text{cmb}} v_a)^2 \sin^2 \theta}}{K} \quad (3.7)$$

We can verify that a minimum value for the first radar detection range will always be defined because the expression under the radical in equation (3.7) is always non-negative and, by definition, $K > 0$. Also, we can see that as K increases (meaning that the interceptor gets faster than the attacker) the minimum value of the first radar detection range needed for valid intercept decreases, what is consistent with the nature of the problem. Hence we can conclude that, in a given scenario, and for a specific value of K , whenever the first radar detection of an arriving attacker occurs at a range shorter than R_{\min} as defined in equation (3.7), using the interceptor in GAI will not make a valid intercept possible. In such case an interceptor on CAP station must be employed.

Now consider scenario like the one in Figure 2. In such a scenario the target is at point T, and is considered again the origin of a cylindric coordinate system. For generality, the air base is located at point B, outside the target area. The CAP is stationed at point C, and at a distance d from the target. In the analysis that follows the following assumptions are made:

1. the radar first detection range (EWR) is deterministic and known;
2. once an attacker is first detected by the early warning radar, the C^3 system is capable of maintaining radar contact with the attacker until either the attacker is intercepted by the interceptor or he attacks at the target;
3. each engagement² is considered as an one-on-one engagement, i.e., each interceptor engages one attacker;
4. the attacker maintains constant air speed through out the raid;
5. the interceptor maintains constant air speed through out the interception procedure;
6. the interceptor can fly over the volume of air space defined by the anti-air weapons effective range only on its way out to CAP station or on his way back to the air base, i.e., he can not fly either through or over this volume during an interception procedure;
7. the identification time, t_{ID} , and the take off time, t_{AI} , are at their minimum values, i.e., these quantities can not be reduced.
8. the attackers' approaching route to the target is not known with certainty, but the angular sector defining all possible approaching route is known with certainty. This angular sector is centered at the target position, and will be referred to as threat sector;

² Here a "engagement" represents the event of an interceptor intercepting an attacker and, if the interception is successful, the air-to-air combat that follows.

9. each raid is equally probable of approaching the target from any direction inside the threat sector, and the probability of having a raid approaching the target by a route outside the threat sector is zero.

The angle θ is formed between the attacker's flight path through the target and the line segment connecting the target position T, with the CAP station position C. This angle is measured from the attacker's flight path into the direction of the CAP station. An attacker is first detected at range R from the target, and because the interceptor is already airborne, there is no delay due to interceptor's take off, so $t_{A1} = 0$, and $t_D = t_{ID}$. To derive an expression for the intercept range in such scenario we apply the same procedure as used to obtain equation (3.2) and obtain

$$S = \frac{K^2(R - t_{ID}v_a) - d\cos\theta}{K^2 - 1} - \frac{\sqrt{K^2(R - t_{ID}v_a)^2 - 2d\cos\theta K^2(R - t_{ID}v_a) + d^2\cos^2\theta + K^2d^2 - d^2}}{K^2 - 1}, \quad (3.8)$$

when $(K > 0) \wedge (K \neq 1)$, and

$$S = \frac{(R - t_{ID}v_a)^2 - d^2}{2[(R - t_{ID}v_a) - d\cos\theta]}, \quad \text{when } (K = 1) \wedge (R \neq d\cos\theta + t_{ID}v_a). \quad (3.9)$$

Under this scenario, the constraints on the values of K becomes

$$K \geq \left| \frac{d\sin\theta}{\sqrt{(R - t_{ID}v_a - d\cos\theta)^2 + d^2\sin^2\theta}} \right| \quad (3.10)$$

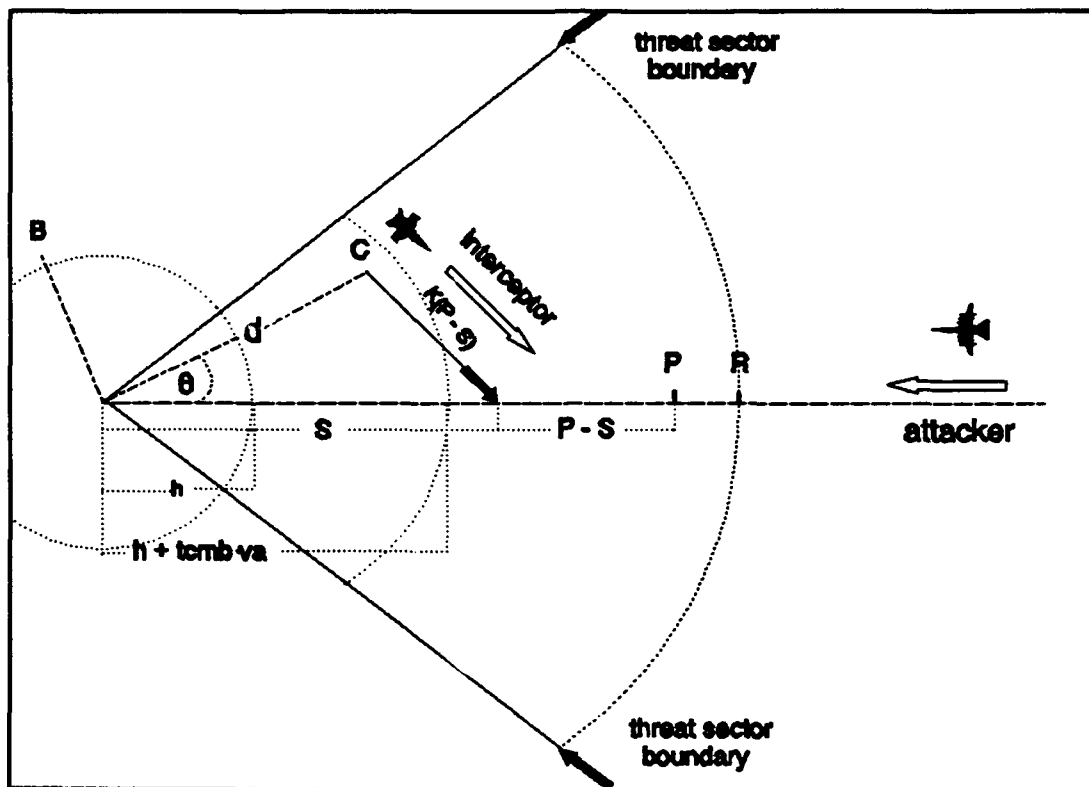


Figure 2 CAP Stationing Scenario

Once the requirement for CAP is identified, the air defense planner must address the issue regarding the defense capability in activate CAP stations. This matter is related with the defense resources availability. To this concern it must be determined how many interceptors are required to activate one CAP station.

C. NUMBER OF AIRCRAFT REQUIRED PER CAP STATION

One factor affecting the maximum number of CAP stations that can be permanently activated is the minimum number of aircraft required to maintain one CAP station activated. Let a represent this number.

Consider the scenario in which one CAP station is activated but no attacker arrives. We will use this scenario to derive an expression for the value of a .

The first interceptor to go to CAP station takes off at time $t = 0$. At time $t = t_{oc}$ this interceptor arrives at station and, because no attacker arrives, it leaves the CAP station at time $t = t_{oc} + t_m$ and flies back to the air base. This means that at time $t = t_{oc} + t_m$ the second interceptor must have arrived on station in order to relieve the first interceptor, what implies that the second interceptor has taken off at time $t = t_{oc} + t_m - t_{oc} = t_m$. This second interceptor stays on station until time $t = t_{oc} + t_m + t_m = t_{oc} + 2t_m$ because no attacker arrives. Hence, at time $t = t_{oc} + 2t_m$ the third interceptor has arrived on station and for this to be possible, the third interceptor has taken off at time $t = t_{oc} + 2t_m - t_{oc} = 2t_m$. Meanwhile, the first interceptor will be ready to take off again to go to CAP station at time $t = t_{oc} + t_m + t_{bc} + t_{rep} + t_{AI}$; the second interceptor will be ready for take off for another sortie at time $t = t_{oc} + 2t_m + t_{bc} + t_{rep} + t_{AI}$; the third interceptor will be ready at time $t = t_{oc} + 3t_m + t_{bc} + t_{rep} + t_{AI}$, and so on. This process will repeat itself until the moment the first attacker arrives. A detailed description of the process is found in Table I below.

Assume in Table I that $t_{oc} > 0$ and $(t_{bc} + t_{rep} + t_{AI}) > 0$. Under these assumptions we have that aircraft number 2 will always be used because

$$(t_{oc} + t_m + t_{bc} + t_{rep} + t_{AI}) > t_m, \forall t_m \geq 0.$$

Aircraft number 3 will be used if and only if

$$(t_{oc} + t_m + t_{bc} + t_{rep} + t_{AI}) > 2t_m, \forall t_m \geq 0.$$

If $(t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al}) \leq 2t_m$, then aircraft number 3 will not be used. To see why this is so notice, in Table I, that aircraft number 3 is supposed to take off at time $t = 2t_m$; if by this time aircraft number 1 is ready to take off again, then number 1 can be launched instead, and aircraft number 3 can be kept on the ground. By the same argument, Table I shows that whenever

$$(t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al}) \leq (a - 1)t_m, \forall t_m \quad (3.11)$$

then aircraft number 1 will be launched instead of aircraft number a . Based on this fact we can use expression (3.11) to determine the value of a as a function of time.

From Table I we have

$$\begin{aligned} (a - 1)t_m &\geq t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al} \\ (a - 1) &\geq \frac{t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al}}{t_m} \\ a &\geq \frac{t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al}}{t_m} + 1, t_m > 0 \end{aligned} \quad (3.12)$$

If we consider that we are interested on the minimum value of a , and that a is integer valued, expression (3.12) becomes

$$a = \left\lceil \frac{t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al}}{t_m} \right\rceil + 1$$

or

Table I

Aircraft#	Take off time	On station time	Hand off time	Ready time
1	0	t_{oc}	$t_{oc} + t_m$	$t_{oc} + t_m + t_{bc} + t_{rep} + t_{Al}$
2	$t_{oc} + t_m$ - $t_{oc} =$ t_m	$t_{oc} + t_m$	$(t_{oc} + t_m) + t_m$ = $t_{oc} + 2t_m$	$t_{oc} + 2t_m + t_{bc} + t_{rep} + t_{Al}$
3	$t_{oc} + 2t_m$ - $t_{oc} =$ $2t_m$	$t_{oc} + 2t_m$	$(t_{oc} + 2t_m) + t_m$ = $t_{oc} + 3t_m$	$t_{oc} + 3t_m + t_{bc} + t_{rep} + t_{Al}$
4	$t_{oc} + 3t_m$ - $t_{oc} =$ $3t_m$	$t_{oc} + 3t_m$	$(t_{oc} + 3t_m) + t_m$ = $t_{oc} + 4t_m$	$t_{oc} + 4t_m + t_{bc} + t_{rep} + t_{Al}$
.
.
.
$a - 1$	$[t_{oc} + (a-2)t_m]$ - $t_{oc} =$ $(a-2)t_m$	$t_{oc} + (a-2)t_m$	$[t_{oc} + (a-2)t_m] + t_m$ = $t_{oc} + (a-1)t_m$	$t_{oc} + (a-1)t_m + t_{bc}$ + $t_{rep} + t_{Al}$
a	$(a-1)t_m$	$t_{oc} + (a-1)t_m$	$[t_{oc} + (a-1)t_m] + t_m$ = $t_{oc} + at_m$	$t_{oc} + at_m + t_{bc} + t_{rep} + t_{Al}$

$$a = \lceil \left(\frac{t_{oc} + t_{bc} + t_{rep} + t_{AI}}{t_m} \right) \rceil + 2, \quad t_m > 0 \quad (3.13)$$

Let A represents the total number of interceptors available in the defense inventory, and let N represents the total number of CAP stations that can be activated. Obviously

$$N = \lceil \left(\frac{A}{a} \right) \rceil \quad (3.14)$$

According to equation (3.14), if a is reduced the defense capability for using the interceptors in CAP is improved.

Equation (3.13) says that the longer an interceptor is capable to stay loitering on station before it has to fly back to the air base, i.e., the bigger t_m is, the fewer the number of aircraft required to maintain one CAP station permanently activated. t_m is directly related to the interceptor endurance characteristics, and one way of improving it by using air refueling. Besides the implications regarding the value of t_m , equation (3.13) also shows that a reduction on the values of t_{oc} , t_{bc} , t_{rep} , and t_{AI} also contributes to decrease a . The repair time, t_{rep} , is a measure of the logistic capability of the Air Interceptor Squadron in terms of Supply and Maintenance. The interceptor reaction time, t_{AI} , reflects the air base infrastructure in supporting the mission. The transit time out to CAP station, t_{oc} , and the transit time back from

CAP station, t_{bc} , are both a function of the CAP station location. These two quantities affect the value of a in two different ways: first because as they increase the numerator in equation (3.13) increases, what makes a bigger; second, because the further away from the air base a CAP station is located, more fuel the interceptor will use to transit back and forth, and this will reduce the value of t_m , making a even bigger. Notice also that if $t_m = 0$, then an infinite number of aircraft would be required to activate one single CAP station, i.e., it would not be possible for the defense to employ the interceptors in CAP.

The above discussion shows that the time variables involved are driving factors in the CAP stationing analysis. To analyze these time variables we are led to the fuel consumption characteristics of the interceptor aircraft.

D. FUEL CONSUMPTION ANALYSIS

The analysis of the scenario in the previous sections shows that for better use of CAP resources one must station CAP to provide optimum conditions for the interceptor to accomplish his mission, while also minimizing aircraft usage.

To provide the optimum conditions for interceptor mission accomplishment from CAP station intercept of an attacker must be completed in such a way that, at the end of the interception, there are time and fuel allowance enough for him to employ his weapons during the air-to-air combat engagement that will follow, maximizing the total probability of obtaining a hit, completed before he is forced to disengage the fight.

To minimize aircraft usage means to place the CAP in a positions relative to the target and air base locations in such a way that the number of aircraft required per CAP station is minimum.

On both of the issues above the fuel consumption plays a key role because it constrains those problem's variables representing time.

1. Maximum Time for Combat (t_{cmb})

The maximum time the interceptor can stay engaged in air-to-air combat with an attacker influences directly the probability of success of the CAP mission.

The probability of success of an engagement aggregates the probabilities of success of many different events. Most of these aggregated probabilities are conditional probabilities by themselves.

As we have seen in Chapter II, Heilenday [Ref. 2] presents a model for the probability of an interceptor to kill an attacker with a single shot. Further he refines the engagement model so that the effects of tactics (type of attack) and technology (type of sensor used in the attack) are also captured by the model. Ball [Ref. 4] gives a plethora of survival models accounting for several different types of threats and encounter environments. The models developed in these studies are a high-resolution type of representation of the probable outcome of the air-air and air-ground encounters. They are applicable to the air-to-air battle scenario whether or not a CAP is considered. So, besides the advantages in fidelity representation, using some of those models in this analysis would not help much in assessing the CAP performance. Considering the purposes of the present study, we will focus on those

aspects of the air-to-air encounter which could hinder the interceptor's capabilities of killing the attacker due to the fact that, prior to the encounter, he was loitering on CAP.

The most significant way the time on CAP impacts the interceptor's capabilities is by diminishing fuel remaining on board at the beginning of the engagement. This is so because the air combat that will follow a successful interception is the phase of the mission where the fuel consumption rate is at its highest and the aircraft is at the furthest distance from the air base.

The outcome of an air-to-air combat is highly dependent on many factors. Some of these factors like training and skill of the pilots involved, the familiarity of each pilot with the opponent's tactics, and level of motivation, for instance, are very hard to represent in an analytical model due to the subjectiveness involved in their quantification. Other factors, besides being suitable for analytical modelling, would add complexity to our investigation without significant contribution to the conclusion, because they are independent of the fact that the aircraft being modelled is on CAP. In what follows we examine some aspects of the air-to-air combat engagement that can be affected by the fact that the aircraft involved was in CAP.

The primary measure of an aircraft's air-to-air combat effectiveness is given by its turn rate. The reason for this is that the turn rate indicates the capability of the aircraft to gain a firing position advantage [Ref. 6:p. 3-24]. Turn rate can be defined as the rate of change of the aircraft's flight direction [Ref. 7:p. 2], and is an intrinsic characteristic of each aircraft. The aircraft's maximum sustained turn rate

capability at a given moment can be determined from its maximum sustained load factor which, by its turn, is obtained from the aircraft's excess specific power at the moment [Ref. 6:p. 3-24]. Expressions for these quantities are derived using energy-state approximation theory and will not be discussed here. However, the interested reader should refer to Shaw [Ref. 1:pp. 388-417], Nicolai [Ref. 6:pp. 3-21 - 3-28], Fellers and Patierno [Ref. 7], or Anderson [Ref. 8:pp. 334-340] for more details. It turns out that the turn rate is affected by the aircraft's gross weight, and the aircraft's weight is affected by the quantity of fuel on board. Furthermore we can use this parameter as a mean to determine the time for combat, t_{cmb} , a quantity connected with the maximum time on station, t_m , and with the a , the minimum number of aircraft required to maintain one CAP station activated.

If aircraft A has a turn rate of ψ_A degrees per second and aircraft B has a turn rate of ψ_B degrees per second, then it is said that aircraft A has a *turn rate margin advantage* over aircraft B of i degrees per second if $\psi_A - \psi_B = i, i > 0$. Usually the air superiority fighter aircraft used as interceptors features better turn rate characteristics than that of the fighter-bomber aircraft normally used for ground attack missions. This is even more so if we consider that an intruder attacker, most likely, is loaded with external ordnance. Hence, it is reasonable to expect an turn rate margin advantage favoring the interceptor (if in a given scenario this is not the case, the planner is cautioned to assess how this fact would impact the overall effectiveness of the interceptors, either on CAP or in GAI, on the air defense mission).

Fellers and Patierno [Ref. 7] show that a desirable turn rate margin advantage over an opponent aircraft is about 2 degrees per second³ [Ref.7:p. 7]. If we consider such an advantage in a head-on engagement, and assuming that the combat evolves only with sustained turns, it would take 90 seconds for the aircraft with the higher turn rate to obtain a definite position advantage in the fight. Here "position advantage" means a spatial displacement between the two aircraft such that the one in advantage is positioned in the rear hemisphere of it's opponent and the noses of both aircraft are pointing in the same general direction. It does not necessarily mean that the aircraft with the advantage has obtained firing conditions. Evidently, an air-to-air combat does not evolve solely based on sustained turns, as other factors such as pilot skill and tactics do interfere. Nevertheless it is reasonable to assume that, if the air combat starts from a head-on engagement at the outset, most likely it will not be terminated before this period of time. On the other hand, experience shows that during an air-to-air combat engagement, as time elapses, if the fight evolves beyond a certain period of time without any definite advantage to either side, then the chances for a subsequent combat advantage definition do not increase if the time of engagement is lengthened. From these considerations we can see that when the combat begins, the shorter the time frame a pilot has to shoot at his opponent, the less probable it is that he will obtain a hit. It is also reasonable to consider that, if we consider the chances of obtaining a hit during an air-to-air combat as a function

³ Measured based on the sustained turn rate, i.e., a turn during which the aircraft does not vary either altitude or air speed.

of the time available for engagement, then this function neither is a linear function nor does it increase monotonically with time. Notice that the time available for engagement can be measured either as a function of distance from the point where the combat starts to a stipulated disengagement point (in this study this is represented by the effective range of the surface anti-air weapons), or as a function of the fuel available on board.

From the above discussion, it is clear that the time for combat, t_{cmb} , is a factor that impacts interceptor's capability of mission accomplishment. According to Fellers and Patierno a "... constant combat time allowance has been standard practice in the past, the magnitude of which has been based on qualitative judgments which have been incorporated into military specifications. It is recognized that determination of the total of fuel required for combat is difficult and somewhat arbitrary. However, for aircraft with varying performance capability, it is more rational to compare them on the basis of providing combat fuel to accomplish a given task rather than requiring fuel for equal combat time." [Ref. 7:p. 6] This way of thinking provides an heuristic for determining t_{cmb} such that the interceptor is granted with the conditions to carry out the fight with the expected probability of success.

First it must be determined the minimum combat time under representative flight conditions, i.e., the minimum time for the interceptor to obtain a 180 degrees gain in direction relative to a typical attacker under the typical expected air battle arena (altitude and air speed ranges during engagement). Dividing 180 by the turn rate margin advantage the interceptor has over the attacker gives an estimate of the

minimum combat time [Ref. 7:p. 7]. To this quantity it must be added the planner's estimated of the typical air-to-air combat time of engagement, i.e., the expected time length beyond which the chances of obtaining a position advantage in a neutral engagement does not improve any more. Usually this quantity is estimated either based on a pre-specified constant number of sustained turns [Ref. 7:p. 6] and translated into a time length , or it can be estimated statistically from field experiments data. Commonly, there is an expected probability of success in the air-to-air combat engagement associated with this number of sustained turns chosen to compute t_{cmb} . This estimated probability is the parameter based on which the planner makes his planning inferences. Next, it must be considered the fuel allowance for a safe disengagement for the case of an unresolved engagement. This can be accounted for by considering an extra number of sustained turns. Summing up these time lengths one obtains a value for t_{cmb} , the maximum time the interceptor can remain engaged in air-to-air combat with the attacker having an expected success probability. Having a value for t_{cmb} , to compute the fuel required for this phase of the interceptor's mission becomes straightforward.

Let p_{kIA}^* be the expected success probability in air-to-air engagement considered by the air defense planning staff. Let t_{cmb}^* represent the time length the interceptor can remain engaged in air-to-air combat with the attacker having expected probability of success in the engagement equals to p_{kIA}^* . The quantity t_{cmb}^* is calculated according to the procedure described above. Define S_{min} as the minimum intercept range for which it is possible for the interceptor to remain

engaged in air-to-air combat for t_{cmb}^* units of time, before he reaches the surface anti-air (AA) effective range line. S_{min} is the minimum distance from the target area that permits the interceptor to begin an air-to-air combat engagement with the attacker with success probability equals to p_{KIA}^* . To compute the value of S_{min} we have to conjecture about the value of v_a during the air-to-air engagement.

During the interception phase of an engagement, the attacker is moving in the target direction at an average ground speed $v = v_a$. When this phase is over one of the following events may take place:

- (1) the attacker keeps flying toward the target area at an average air speed $v = v_1$, $v_1 = v_a$, because the interception is not successful; or,
- (2) the interception is successful and the attacker start maneuvering to evade the interceptor but keep moving toward the target area at an average air speed $v = v_2$, $v_2 < v_a$; or,
- (3) the interception is successful, and the attacker engages air-to-air combat with the interceptor; in this case, provided he is alive, his movement toward the target area has an average speed $v = v_3$, $0 \leq v_3 < v_a$.

If we consider all the randomness present in the process that an air-to-air engagement represents, it becomes clear that to predict or estimate values for v_2 or for v_3 is not a simple task. Keeping in mind that we are seeking for a value for S_{min} , an underestimation of v may lead to a value for S_{min} closer to the target than the actual distance required for the interceptor to have time for combat enough to have p_{KIA}^* chances of success. If $v = v_a$ is used instead, then an overestimation is being made, and the defense problem becomes tighter. However, by doing so, this ensures that the interceptor can perform with p_{KIA}^* chance of success, and the air defense

planner reduces the risk of allowing the interceptor to fly into the hazardous firing area protected by surface AA weapons. Hence, it is reasonable to use a value $v = v_a$ to compute S_{min} . Thus we get:

$$S_{min} = h + t_{cmb}^* \cdot v_a \quad (3.15)$$

As it was pointed out before, there are many different factors that affect the probable outcome of an air-to-air combat engagement, including psychological aspects as well as time/fuel constraints. However, if the air defense planner does study the scenario and the opponent's characteristics carefully, the result will be an adequate value for t_{cmb}^* and the equivalent combat fuel apportionment such that the interceptor engaging from CAP is granted with the optimum conditions for mission accomplishment.

2. Maximum Time on Cap Station (t_m)

As equation (3.13) shows, t_m , the maximum time an interceptor can stay loitering on station if no attacker arrives, is a key factor to determine aircraft usage in CAP. It is clear that t_m depends on the fuel available to the interceptor at the moment he arrives on station. To estimate how long the interceptor can stay loitering on CAP if no attacker arrives, it have to be analyzed how much fuel he must have on board at the moment he leaves CAP station to fly back to the air base. This amount of fuel is sometimes referred to as the "combat package".

Lets assume that exactly at the moment the interceptor is leaving CAP station an attacker arrives and there is no other interceptor available. Then this interceptor,

who is just about to leave the CAP, will have to engage the arriving attacker. To make this interception possible the interceptor must have, at this moment, fuel enough to intercept the attacker, to sustain air-to-air combat for t_{cmb} units of time and to fly back to the air base with a safe amount of fuel. If we sum up all the fuel consumed from the moment this hypothetical engagement started until the interceptor landing in the air base, we have an estimate of the minimum amount of fuel the interceptor have to have on board when he leaves CAP if no attacker arrives. Subtracting this quantity from the amount of fuel he has on board at the moment he arrives on station gives the value of f_{CAP} , the amount of fuel for loitering on station before having to fly back to the air base. Having a value for f_{CAP} a value for t_m can be determined straightforward by considering the aircraft maximum endurance schedule for fuel consumption setting.

There is one aspect in the above discussion that have to be addressed more carefully, namely t_{intc} . Besides not being represented in equation (3.11) it is worth analyzing because it affects t_m .

It is clear that t_{intc} depends on the value of K , i.e., the interceptor/attacker speed relationship. Equations (3.8) and (3.9) show the role K plays in the interception geometry. It worth emphasizing that in those equations K represents a ratio of average air speeds. Usually, if there is no threat in the scene, the interceptor loiters on CAP station at or around the maximum endurance air speed for the loitering altitude. This air speed is, most often, well below those speed setting used during the intercept procedure. At the beginning of the interception an acceleration

to a selected final intercept air speed will take place. Depending on the value of this air speed selected, the power setting required to accelerate may cause a very high fuel consumption rate what would imply in a greater amount of fuel being apportioned to the interception phase. Because the air-to-air combat engagement is the paramount phase of the entire mission, it is not prudent to consume the fuel for combat during the intercept phase. Consequently, a high fuel consumption during the intercept will cause a reduction in the fuel for loitering hence abbreviating t_m and increasing the value of α .

The impact of the intercept air speed on the intercept range was investigated by Fellers and Patierno [Ref. 7]. To this end a typical situation was hypothesized in which the interceptor is engaged from GAI with a scramble time of 3 minutes instead of being engaged from CAP; an intruder flying at an air speed equals to Mach 0.9, and a first radar detection range of 200 nautical miles is considered. This difference in scenario does not invalidate the results for the present study. It was shown that, if a tactical environment is considered, i.e., a first radar detection range is somewhat limited, the intercept distance is relatively unaffected by maximum speed capabilities greater than Mach 2.0. In Figure 3 below the data obtained in this investigation are reproduced. [Ref. 7:p. 5]

Usually the intercept speed is chosen based on the attacker's speed. One criterion for such a choice is to set the interceptor's air speed to values that vary from 1.2 times the attacker's air speed up to the interceptor's maximum speed limit [Ref. 9:p. 4-16]. But, as Figure 3 shows, a high setting of final intercept speed is of

little value for the tactical intercept mission, in addition to the fact that it increases substantially the fuel consumption rate. Hence, in order to increase t_m , it is necessary to station the CAP in such a way that it is possible to intercept an attacker without having to resort to high values of K , mainly in the cases when the interceptor is at the limit of fuel allowance. As a heuristic for selecting a value for K in a given scenario, expression (3.10) gives the conditions which makes the an intercept possible.

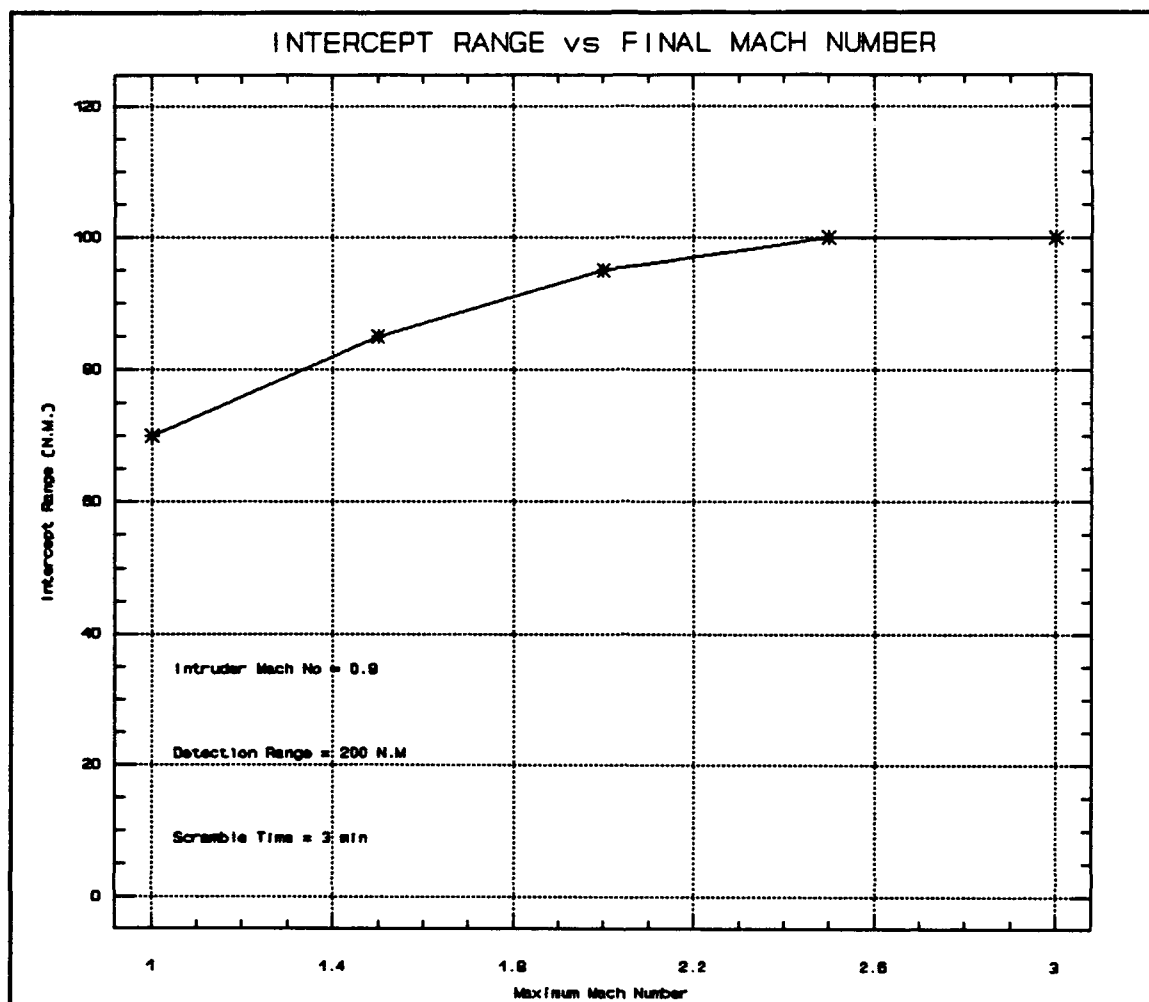


Figure 3 Intercept Range as a Function of the Intercept Maximum Air Speed [Ref. 9:p. 5]

IV. A MACRO MODEL FOR CAP MANAGEMENT

The maintenance of a CAP requires the management of various resources so as to achieve optimal attrition of the attackers. The defending force will have an initial number of interceptors, each with certain endurance and combat capability. In addition, the aircraft will be supported logistically by a repair facility at their base. Options open to the defence force commander are, among others,

- (1) to choose the distance at which to station the CAP from target (and from support air base);
- (2) to select the nominal maximum number of aircraft on CAP;
- (3) to decide upon the altitude at which CAP operates, and that at which the intercept will be made.

All of the above choices are influenced by the endurance of the aircraft, and by their fuel consumption characteristics. A realistic choice of CAP size and location must be influenced by the realities of aircraft endurance, by the system expected kill probability in each engagement, and by the necessity to provide ground turnaround to an aircraft after it returns from a CAP mission, whether it be merely a patrol or involve actual interception and combat.

In order to shed light on the overall CAP performance a simplified macro model will now be described. After the various states of aircraft engaged in the CAP operation are described important parameters that control transition between states

are defined, and the deterministic state transition equations are presented. Numerical solutions illustrate the system behavior.

A. STATES OF THE SYSTEM

The behavior of an interceptor aircraft during an air defense campaign employing CAP can be described as a dynamic system. In such a system, at any point in time, an alive interceptor is found in one of several different states. For simplicity we will represent here only the states which, in one way or another, could have some impact in the options open for the decision maker. Hence we consider that during the campaign, an aircraft allocated for the CAP role can be found in one of the following states:

- (1) on the ground in the air base, ready to be launched to CAP station;
- (2) flying out from the air base, going to CAP station;
- (3) loitering on station;
- (4) engaged in an intercept/air-to-air combat with the attacker;
- (5) flying back to the air base, after terminating an intercept/air-to-air combat with the attacker, or from CAP station if has not been engaged in any attacker interception;
- (6) on the ground in the air base, being serviced in the air base repair facility.

Once the states in which a surviving interceptor can be found at any time t are identified, we proceed to define the state variables of the system.

B. STATE VARIABLES DEFINITION

In general, one way by which a macro model is a helpful tool for analysis purposes is that it is simple to design so as to keep track of the history in time of the changes in the state variables. Therefore, in order to have a snapshot of the state of the system at any moment, we define the state variables as a function of time as follows

$N_G(t)$ = number of interceptors on ground alert at time t .

$N_{OC}(t)$ = number of interceptors in transit out to CAP station at time t .

$N_C(t)$ = number of interceptors in CAP at time t .

$C1(t)$ = number of interceptors engaged from CAP station in intercept/air-to-air combat with an attacker at time t .

$C2(t)$ = number of interceptors engaged from their way out to CAP station in intercept/air-to-air combat with an attacker at time t .

$N_B(t)$ = number of interceptors flying back to air base at time t .

$N_R(t)$ = number of interceptors being repaired at the air base repair shop at time t .

The variables defined above only make sense if they are framed by the defense's interceptors inventory, otherwise they would be unbounded. So we define:

A = total number of interceptors in the defense inventory at the beginning of the campaign.

$A(t)$ = number of interceptors alive at time t ; there can, in principle be attrition during combat, so, unless reinforcement are possible, $A(t) \leq A$.

From the definitions above it follows that

$$A(t) = N_G(t) + N_{OC}(t) + N_C(t) + C1(t) + C2(t) + N_B(t) + N_R(t) \quad (4.1)$$

A diagrammatic representation of the macro model for CAP management is presented in Figure 4 below, where a block diagram shows the possible interactions between different states of the system.

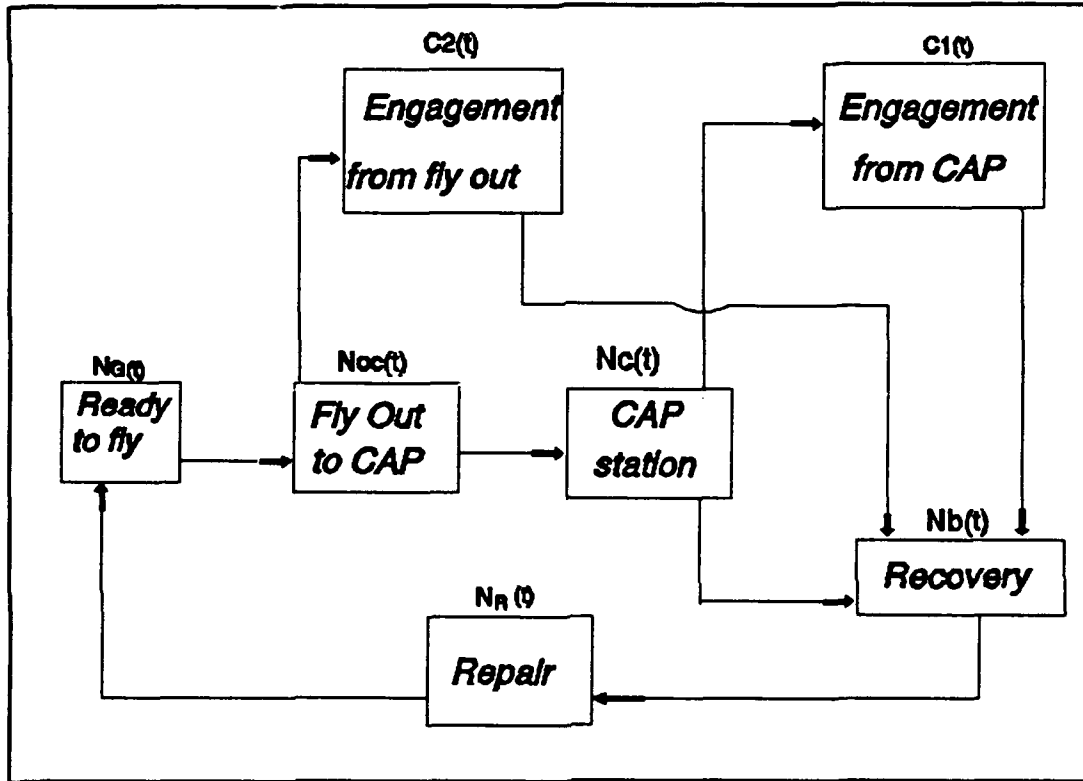


Figure 4. State variables interactions

The idea underlying this model is to trace dynamically the changes in the levels of the state variables, given the initial conditions of each variable at some initial time t_0 . Consequently we must define the parameters that control transition between each pair of interacting states.

C. PARAMETERS OF THE SYSTEM

The flow of aircraft from one state to another is governed by the system's parameters. Each of these parameters represent the *rate* at which aircraft leave each state of the system. Hence the unit of measure of each the parameters is *aircraft per unit of time*.

In order to simplify the representation of the interactions present in a complex process such as the one we are modeling, we will consider the state variables as deterministic. By suppressing the randomness, the model may not portraint all details and constraints otherwise captured if a high-resolution type of combat simulation were adopted. However, for air defense planning and CAP stationing analysis the technique used here can provide useful guidance for the decision maker.

The parameters are computed based on the following assumptions regarding the scenario modeled:

- (1) the attacker is equally likely to approach the target area from any direction within the threat sector;
- (2) the attacker is flying at constant altitude and air speed, and in the target direction;
- (3) all attackers represent the same level of threat for the defense;
- (4) once the defense's C³ system first detects the attacker, there is no loss of radar contact through out the engagement;
- (5) each engagement is considered as a one-on-one encounter, and in each encounter the interceptor uses all its ammunition;
- (6) the attackers feature lethal self defense capability;

(7) the defense tries to maintain a force size on CAP equal to \bar{c} interceptors at a time. Note that \bar{c} is a decision variable, the magnitude of which affects CAP sustainability and hence effectiveness.

The parameters' values are derived based on the time variables and fuel variables defined in the previous chapter. All parameters are computed based on the average value of the variables involved. So, the computation of some of those variables must precede the state variables and MOE computations.

1. Time and Fuel Variables

For a given scenario the average values of the following variables must be determined before the parameters can be computed:

- (1) t_{oc} , time length it takes to the interceptor to fly from the air base out to the CAP station;
- (2) f_{oc} , the amount of fuel used by the interceptor to fly from the air base out to the CAP station;
- (3) t_{bc} , time length it takes to the interceptor to fly from CAP station back to the air base;
- (4) t_{intc} . Because the intercept distance for an interceptor engaging an attacker from the CAP station is different from that of an interceptor engaging from his way out to CAP, t_{intc} have to be computed separately for each case;
- (5) f_{intc} , the amount of fuel used during an interception;
- (6) t_{be} , time length it takes to an interceptor to fly from the combat disengagement point back to the air base;
- (7) f_{be} , fuel amount used by the interceptor to fly from the combat disengagement point back to the air base;
- (8) t_m , maximum length of time an interceptor can stay loitering on station if no attacker arrives;

(9) f_{CAP} maximum amount of fuel an interceptor can use for loitering on station if no attacker arrives;

In order to compute the average values of the time variables involved, a scenario such as the one depicted in Figure 5 is considered. In this scenario the air base is located at point B, and the target is at point T; the CAP is stationed at point C, at a distance d from the target; there is a radar sensor located in the target area with an expected radar first detection range equals to R .

Before we proceed computing the (mean) values for the time variables, recall that θ is the angle formed by the attacker's flight path passing through the target location (T) and the line segment connecting the target location and the air base location (B); $\theta \geq 0$. In what follows the expression "flight parameters" stands for combination of the aircraft gross weight, the flight altitude and the aircraft's drag characteristics at the moment considered.

Values of t_{oc} and f_{oc} : the values of t_{oc} and f_{oc} are computed based on the distance from air base to CAP station, and considering the long range schedule of air speed and fuel consumption settings for the interceptor aircraft based on its current flight parameters during this transit out phase. The value of t_{bc} in the same fashion, but considering the current flight parameters of the interceptor aircraft during the transit back phase of the flight.

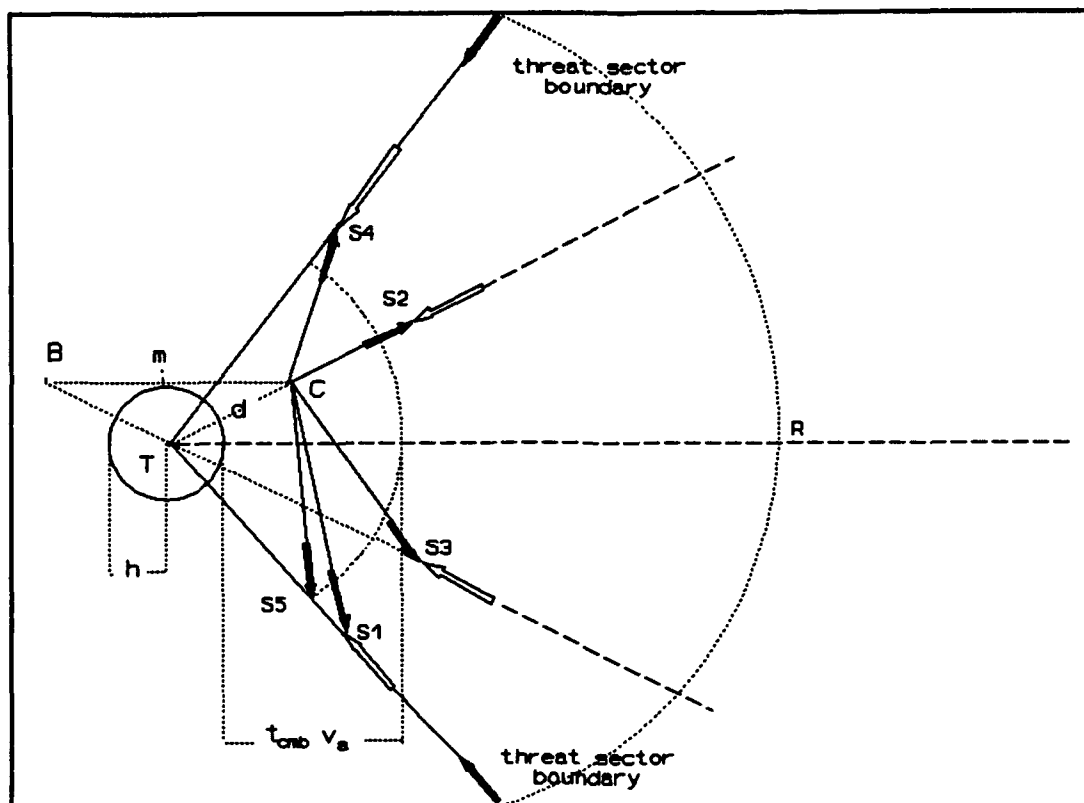


Figure 5 CAP scenario

Mean values of t_{intc} and f_{intc} : to compute the mean value of the intercept time and fuel for the case when the interceptor is engaged from CAP, we proceed as follows:

- (1) Determine the value of θ which yields the longest distance of intercept (d_{intc}) as measured from the CAP position.
- (2) The value of θ which yields the shortest d_{intc} will occur when θ is equal to 0.
- (3) Using equation (3.8) or (3.9), determine the values of the longest and shortest d_{intc} . In Figure 5 these values occur when the attacker is intercepted, respectively at points S_1 and S_2 .
- (4) Determine the largest and the smallest values for t_{intc} using S_1 , S_2 , and according the fuel consumption setting required for an air speed determined by the values of K and v_s ; the mean value of t_{intc} is the average of these values.

(5) The mean value of f_{intc} is computed considering the schedule of fuel consumption setting used above, and considering the mean t_{intc} .

For the case in which the interceptor is engaged from his way out to CAP station, we need to determine the mean distance from the air base to CAP station, what in Figure 5 is represented by the point m. The computations are analogous to the above, with the difference that the shortest d_{intc} will always occur when an attacker flying along the threat sector boundary that is positioned in the same hemisphere as the air base position is intercepted.

Mean values of t_{be} and f_{be} : to compute the average recovering time and equivalent fuel amount, we have to determine those values of S which yields the longest and the shortest recovery distances (d_{rec}). Obviously, the longest and shortest recovering distance are measured from the air base position (point B in Figure 5) to the closest and furthest intercept points. In Figure 5 these points are represented, respectively, by the points S_3 and S_4 . These values are the same either the interceptor is engaged from CAP station or not. To compute t_{be} use the fact that the angle formed between the line segment connecting point B to point T and the bisector of the threat sector is known. Then we proceed as follows:

- (1) Determine the value of θ for the shortest d_{rec} . Notice that the shortest d_{rec} will occur when an attacker flying along the threat sector boundary that is positioned in the same hemisphere as the air base position is intercepted.
- (2) The value of θ for the longest d_{rec} depends on the displacement of the air base with respect to the target position and the threat sector. This value will be either 0 or it will be the one determined by the interception of an attacker who is flying along the threat sector boundary in the opposite hemisphere then the air base.
- (3) Using equations (3.8) or (3.9) determine the values of S_3 and S_4 .

(4) Determine the smallest and the largest values for d_{rec} . The mean value for d_{rec} is the average of the values just found.

(5) t_{be} and f_{be} are then computed using the mean value for d_{rec} , and considering the long range schedule of air speed and fuel consumption settings for the interceptor aircraft based on its current flight parameters during this transit back phase.

Values of t_{cmb} and f_{cmb} : to compute t_{cmb} and f_{cmb} , we do steps one through three as in the procedure to compute t_{intc} , then procedure as follows:

(1) Compute

$$t_{cmb} = \begin{cases} 0, & \text{if } S < h, \quad S = \min\{S_1, S_2\} \\ \frac{S - h}{v_a}, & \text{if } S \geq h, \quad S = \min\{S_1, S_2\} \end{cases} \quad (4.2)$$

Notice that when $S < h$ the interceptor can not engage air-to-air combat because the attacker is within the anti-air weapons range already. In such case the intercept is of no value for the defense.

(2) Compute f_{cmb} considering the combat performance schedule of fuel consumption setting for the interceptor aircraft.

Value of t_m : to compute the value for the maximum length of time the interceptor can stay on station if no attacker arrives, we need to address the issue of interceptor's fuel consumption in each phase of his mission. In doing so we proceed according to the following steps:

(1) Determine f_{oc} , the amount of fuel used by the interceptor to fly out from the air base to CAP station. To this end one considers the long range schedule of air speed and fuel consumption settings for the interceptor aircraft based on its flight parameters during this transit out phase. The value of f_{oc} is affected by factors as: the distance from the air base to CAP station

(2) Determine the furthest point from the CAP position where a valid intercept is possible, and determine d_{intc} for this point. This quantity is affected by the CAP station location within the scenario and by the selection of K , the interceptor/attacker air speed relationship. In Figure 5 such a point is represented by S_5 .

(3) Compute f_{intc} , the fuel required to intercept the attacker at S_5 when the interceptor is on the CAP station at the begins the interception. f_{intc} is influenced by both, the CAP station position and the selection of K .

(4) Determine the d_{rec} which corresponds to this point, and compute f_{rec} , the fuel required to fly from the intercept point back to the air base. The value of f_{rec} is computed considering the long range schedule of air speed and fuel consumption settings for the interceptor aircraft based on its flight parameters during this transit back phase.

(5) Compute f_{CAP} , the maximum fuel available for loitering at the CAP station as follows

$$f_{CAP} = f_{TOT} - (f_{oc} + f_{intc} + f_{cmb} + f_{rec} + f_{safe}). \quad (4.3)$$

(6) Given f_{CAP} , compute t_m based on the maximum endurance schedule of air speed and fuel consumption settings for the interceptor aircraft based on its average gross weight during this loitering phase.

(7) From the previous computations we are able to compute the average value of t_{bc} , the time it takes to the interceptor to fly from the CAP station back to the air base. t_{bc} is computed considering the distance from CAP station to the base, and using the long range schedule of air speed and fuel consumption settings for the interceptor aircraft considering an average gross weight based on the fuel remaining on board at the beginning of the transit back. This amount of fuel is estimated as $f_{TOT} - (f_{oc} + f_{CAP})$.

2. Parameters Definition

Having defined the values of the time parameters we now define the rate parameters of the deterministic macro model for CAP management. Each parameter used in the model is defined as below.

λ_{AI} = rate at which interceptors on ground alert take off when they are scrambled.

λ_{AI} represents the ground support for the mission. The value of λ_{AI} is influenced by factors such as taxi way characteristics in the air base, availability of Quick Reaction Alert (QRA) facilities in the air base, availability of ground equipment and ground crew to assist multiple aircraft start up, etc. λ_{AI} is computed as

$$\lambda_{AI} = \frac{1}{t_{AI}} \text{ aircraft per unit of time}$$

The parameters defined next give a measure of the aircraft fuel consumption characteristic in each phase of a CAP mission; all of them are influenced by the CAP positioning relative to the target and air base sites as well.

λ_{oc} = rate at which interceptors finish the transit out to CAP station when they are

$$\lambda_{oc} = \frac{1}{t_{oc}} \text{ aircraft per unit of time}$$

λ_{bc} = rate at which interceptors finish their transit back to the air base when they fly from CAP station to the air base due to not have being engaged in the interception of any attacker.

$$\lambda_{bc} = \frac{1}{t_{bc}} \text{ aircraft per unit of time}$$

λ_{be} = rate at which interceptors finish their transit back from engagement when they conclude a mission and fly from the point where the combat is terminated back to the air base.

λ_{cb} = rate at which interceptors leave CAP station if they are not engaged.

$$\lambda_{be} = \frac{1}{t_{be}} \text{ aircraft per unit of time}$$

$$\lambda_{cb} = \frac{1}{t_m} \text{ aircraft per unit of time}$$

The next parameter, λ_a , is a measure of the attacking air force capabilities. Its value drives the whole dynamic process represented in the model, hence is a key factor for modelling considerations and output analysis. One way to determine a value for this parameter is by means of situation assessment or intelligence report analysis. Its value can be taken as being constant or as being time dependent, where t_a variable.

λ_a = rate at which attacker aircraft enter the defense radar coverage.

$$\lambda_a = \frac{1}{t_a} \text{ aircraft per unit of time}$$

The next parameter, λ_R , represents the logistic support provided by the air base to the CAP mission in terms of maintenance and spare parts supply. Needless to say that these two factors may represent serious constraints to air operations of any kind. To capture all facets of the logistic process specifically supporting the CAP mission requires a separate study by itself. In this sense, λ_R is a simplified surrogate for such process.

t_{rep} = time length it takes for one repair crew to repair one aircraft.

c_R = number of repair crews available to service interceptor aircraft in the air base's repair shop.

λ_R = rate at which aircraft leave the air base repair shop facility after being serviced.

$$\lambda_R = \begin{cases} 0 & \text{aircraft per unit of time, if } N_R(t) = 0; \\ \frac{N_R(t)}{t_{rep}} & \text{aircraft per unit of time, if } 0 < N_R(t) < c_R, t_{rep} > 0; \\ \frac{c_R}{t_{rep}} & \text{aircraft per unit of time, if } N_R(t) \geq c_R, t_{rep} > 0. \end{cases}$$

3. The Attrition Rate

The attrition rate portrays the rate at which the weapon-systems being modeled do inflict or sustain casualties. Any model in which some kind of combat process is represented becomes very sensitive to the way attrition is modelled. Taylor [Ref. 10] presents a comprehensive study on this regard with a ground battle orientation.

Two approaches are usually adopted for the numerical determination of the attrition rate: (a) a statistical estimates based on "combat" data generated by a detailed Monte Carlo combat simulation; or (b) an analytical submodel of the attrition process for the particular combination of firer and target types. The first method uses the output of a Monte Carlo simulation "...to fit one or more free parameters in the analytical model so that it will at least duplicate and hopefully predict results comparable to those obtainable from the simulation model." [Ref 11:p. 45] The conceptual idea for the second approach is to develop an analytical

expression for each attrition-rate coefficient used in the model by considering a single firer engaging a passive target, i.e., one that does not fire back. [Ref. 10:pp. 45-46]

According to Taylor, a general methodology for determining analytically the attrition-rate coefficients for a wide spectrum of weapon-system types engaging specified target types was developed by Bonder and Farrell. The idea underlying the method is to take the attrition-rate coefficient as being the reciprocal of the expected time for an single firer to kill a single target. Hence, if we let k represent the attrition-rate, the Bonder and Farrell methodology gives

$$k = \frac{1}{E[T_{XY}]},$$

where

T_{XY} is a random variable denoting the time required for an individual Y firer to kill a single X target; and,

$E[T_{XY}]$ is the expected value of the random variable T_{XY} . [Ref. 10:pp. 47-48]

To compute the expected value above, the weapon-systems were classified, according to the lethality mechanism characteristics, as being either damage-by-impact or damage-by-area type of weapon. Within each category, the weapon-systems were also classified according to the firing doctrine employed, i.e., how is firing information used to control the weapon-system's aim point and its delivery characteristics. Based on this it was concluded that there is a large class of weapon-systems that can be seen as a Markov-dependent-fire weapon, i.e., the outcome of the firing of a round by the weapon-system depends only on the outcome of the

immediately preceding round. An expression for the expected value of the time it takes to such weapon to kill a target was developed. The reader is referred to Taylor [Ref. 10:pp. 48-51] for more details about the methodology and the expression for the expected value. For details on the derivation of that expression, see Hartman [Ref. 11:pp. 129-135].

For the macro model for CAP management we will use a rather simple attrition model based on the idea above of taking the attrition rate as the reciprocal of the expected value of the time to kill a target. In this context the target will be an attacker or an interceptor, depending on which attrition is being considered.

When one aircraft is firing at his opponent during an one-on-one air-to-air combat engagement the opponent, most of the time either does not have spatial displacement to fire back, or he is not aware of the firer aircraft presence in the scene. In either case we have the situation in which one firer is firing at a passive target, i.e., a target that does not fire back. This may not be true if we consider a one-on-many or a many-on-many type of engagements but, as we are assuming in our macro model that each engagement is an one-on-one encounter, we can consider in this study that each firer is firing at a passive target. To compute the expected value of the time it takes for an interceptor in CAP to kill or neutralize an attacker we reason as follow.

Let p_{intc} represent the expected probability that an interceptor engaged from CAP station successfully intercepts an attacker, where the value of p_{intc} was obtained from field experiments. Consider p_{kIA}^* and t_{cmb}^* as defined in Chapter III.

Define P_{kIA}^* as the probability that an CAP station interceptor either kills or neutralizes the attacker given that he has fuel to remain engaged in air-to-air combat for t_{cmb}^* units of time. Clearly

$$P_{kIA}^* = p_{intc}^* \cdot P_{kIA}^*$$

Each intercept trial can be seen as a Bernoulli trial having probability of success equals to P_{kIA}^* . Under the assumption that on each encounter the interceptor uses all his ammunition (what is a reasonable assumption depending on the aircraft considered), we have that one interceptor is capable of performing a single engagement, and hence each intercept trial will be accomplished by a different interceptor. Based on this we have that each intercept attempt is an independent Bernoulli trial.

To simplify the explanation, in what follows it is assumed that each intercept trial is performed by a different interceptor, and that each interceptor is engaged from CAP station and has fuel enough to remain engaged in air-to-air combat for t_{cmb}^* units of time.

Let T , $T = 1, 2, 3, \dots$, be the number of interception attempts needed upon one attacker to either kill or neutralize the attacker. Clearly T is a random variable with geometric distribution and parameter P_{kIA}^* . Denote I , $I = 1, 2, 3, \dots$, as the average number of intercept trials required to either kill or neutralize one attacker. Then, from probability theory, I is the expected number of intercept trials to either kill or neutralize an attacker and can be expressed as

$$I = E[T] = \frac{1}{P_{kIA}^*}, \quad 0 \leq P_{kIA}^* \leq 1 \text{ and } T = 1, 2, 3, \dots \quad (4.4)$$

Consider that there are infinite many interceptors on CAP station, and assume that when each of these interceptors operates combined with the C^3 system, they both make up a perfect weapon-system, i.e., a weapon-system for which $P_{kIA}^* = 1$. Then only one intercept trial will be required to kill or neutralize each attacker who arrives. Furthermore, it will take $t_a + t_{ID} + t_{intc} + t_{cmb}^*$ units of time for such perfect weapon system to kill the first arriving attacker, and this will be the time interval between each attacker's casualties. Obviously such a system does not exist, and maintaining infinite aircraft on CAP station is not realistic either. Nevertheless, such hypothetical scenario shows that, considering the realities constraints represented by equation (4.4), in the average, I trials are required to obtain a kill. Consequently, the defense decision maker can use the value of I as an heuristic to decide on the minimum force size on CAP. Let \bar{c}^* , $\bar{c}^* = 1, 2, 3, \dots$ be the required number of interceptors to be kept on CAP per expected attacker. The value of \bar{c}^* is determined as follows:

$$\bar{c}^* = I = \lceil \left\lfloor \frac{1}{P_k^*} \right\rfloor \rceil = \lceil \left\lfloor \frac{1}{P_k^* \cdot P_{intc}^*} \right\rfloor \rceil \quad (4.5)$$

Let T_{kIA}^* represent the average length of time between two consecutive attackers's casualties. Then, from the discussion above, we have
and from this equation we obtain

$$T_{kIA}^* = I \cdot (t_a + t_{ID} + t_{intc} + t_{cmb}^*) = \frac{t_a + t_{ID} + t_{intc} + t_{cmb}^*}{P_{kIA}^*}$$

$$T_{kIA}^* = \frac{t_a + t_{ID} + t_{intc} + t_{cmb}^*}{P_{intc}^* \cdot P_{kIA}^*}, \quad 0 < P_{intc}^* < 1 \quad \text{and} \quad 0 < P_{kIA}^* < 1 \quad (4.6)$$

If we consider that P_{kIA}^* and P_{intc}^* are surrogates for the efficiency of the defense's C³ system and interceptor synergism, equation (4.6) makes sense. It says that the more efficient the weapons-system, the shorter the average time to inflict a casualty. So, any degradation on the values of P_{kIA}^* and P_{intc}^* implies on a reduction of the CAP efficiency in terms of attackers killed per unit of time. Hence, before we can compute the attrition rate for the macro model in a given scenario, any degradation on the values of P_{kIA}^* and P_{intc}^* due to scenario realities must be identified and accounted for in the model.

One way P_{intc}^* can be degraded is due to Electronic Counter Measures (ECM) from the part of the attackers. If the attackers do use ECM, the CAP performance is affected no matter what the CAP station position is. So this type of degradation will not be treated in this study. Regarding P_{kIA}^* , it is clear from equation (3.15) and from the rationale that precedes it, that the actual range from the target area an attacker is intercepted does affect the interceptor's chances of success during the air-to-air combat that follows the interception.

Define T_{kIA} as the average length of time between two consecutive attackers's casualties in a given scenario. Let p_{kIA} represent the success probability in air-to-air

combat engagement the interceptor has in a given scenario; let p_{intc} represent the probability that an interceptor engaged from CAP station in a given scenario successfully intercepts an attacker. Considering the purposes of the present study, we will let $p_{intc} = p_{intc}^*$. To correct the value of p_{kIA}^* for the scenario realities, the value of t_{cmb} is determined using equation (4.2), and the value of p_{kIA} is computed as follows

$$p_{kIA} = \begin{cases} p_{kIA}^*, & \text{if } t_{cmb} \geq t_{cmb}^* \\ p_{kIA}^* \cdot \left(\frac{t_{cmb}}{t_{cmb}^*} \right)^4, & \text{if } 0 < t_{cmb} < t_{cmb}^* \\ 0, & \text{if } t_{cmb} = 0 \end{cases} \quad (4.7)$$

Equation (4.7) degrades substantially the interceptor's chances of success in an air-to-air engagement if at the end of the interception the time available for combat is below the value that grants the expected conditions for mission accomplishment. Consistent with the argument in Chapter III, Section , Subsection , the fourth power in this equation says that, regarding the chances of success in the air-to-air combat, to use the fuel for combat to any other phase of the mission will degrade the CAP interceptor's mission performance. In Figure 6 we have a plot of the probability of kill in air-to-air combat as a function of the time available for combat. This Figure is based on equation (4.7), when considering a typical tactical scenario with a radar first detection range of 75 nautical miles measured from the target position; an anti-air weapons effective range of 10 nautical miles around the target area; the value for

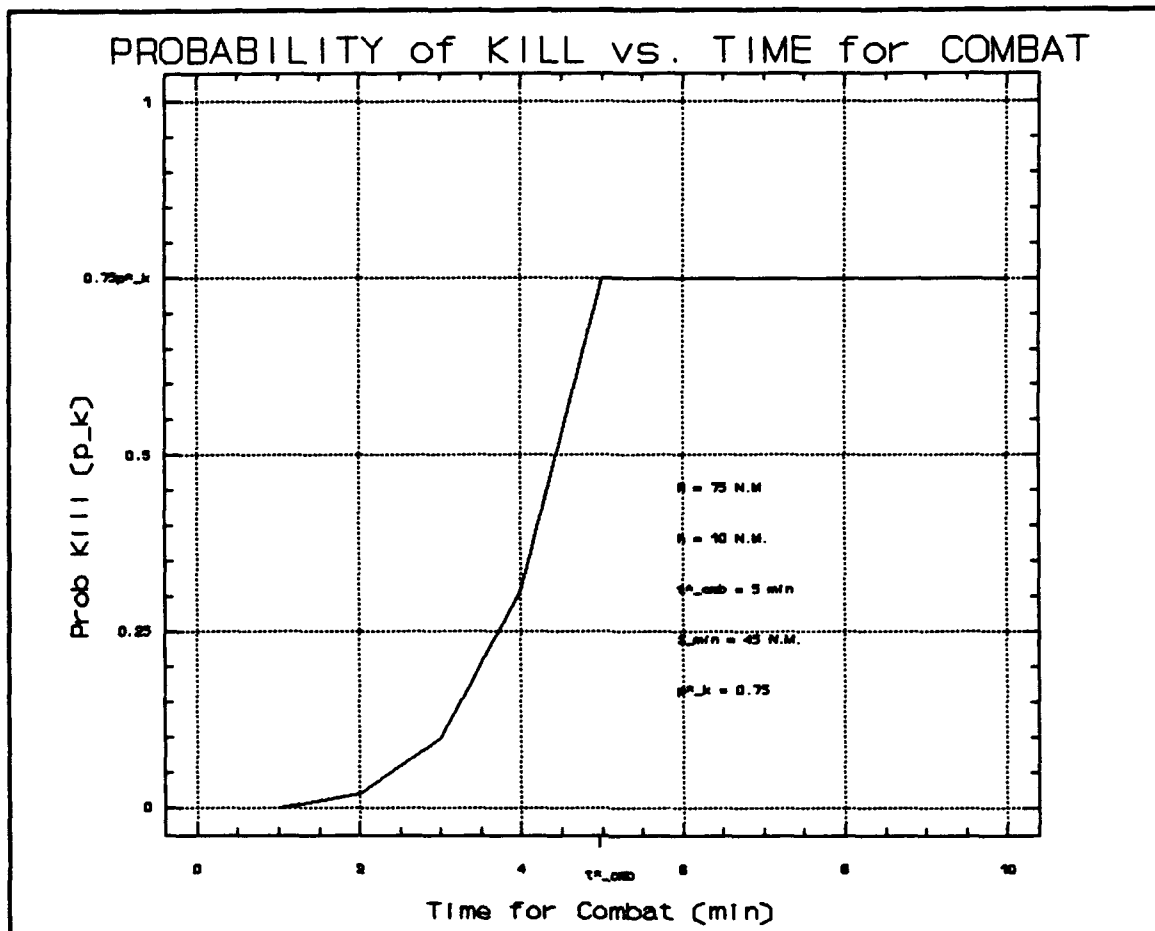


Figure 6. Probability of Kill as a function of the time available for combat

p_{kIA}^* is 0.75, with a t_{cmb}^* of 5 minutes. Considering an attacker flying at an air speed equals to Mach 0.7 equation (3.15) gives a value of S_{min} equals to 45 nautical miles.

Once the scenario effects has been captured, the expected time between two consecutive attackers casualties can be computed. Let T_{kIA} represent this quantity. The expression for this expected value is obtained rewriting equation (4.6) as following

$$T_{kIA} = \begin{cases} \frac{t_a + t_{ID} + t_{intc} + t_{cmb}}{p_{intc} \cdot p_{kIA}}, & \text{if } 0 < p_{intc} < 1 \text{ and } 0 < p_{kIA} < 1 \\ 0, & \text{if } p_{kIA} = 0 \text{ or } p_{intc} = 0 \end{cases} \quad (4.8)$$

where p_{kIA} is computed using equation (4.7) above, t_{cmb} is computed using equation (4.2), and the value of p_{intc} is equal to p_{intc}^* . Notice that when either p_{kIA} or p_{intc} is equal to 0, then no casualty is inflicted to the attackers, meaning that the air-to-air combat engagement is not possible in the given scenario.

Notice in equation (4.8) that the expected time between casualties is a function of t_{intc} . If the possibility of a transient interceptor⁴ to engage an interception is considered, then there will exist two different values for t_{intc} in the scenario, yielding distinct values for T_{kIA} . So, there must exist two distinct variables in the model to account for this situation. Hence, let $T1_{kIA}$ be the expected time between two consecutive casualties inflicted to the attackers by the interceptions starting from CAP station; and $T2_{kIA}$ be the expected time between two consecutive casualties inflicted to the attackers by transient interceptors. Additionally, define $t1_{intc}$ as being the time elapsed from the moment an interception beginning from CAP station starts until the moment the interceptor engages air-to-air combat with the attacker; and $t2_{intc}$ as being the analogous variable for the case in which the interception is performed by transient interceptor. Then

⁴ Here a "transient interceptor" means the interceptor who is flying out from the air base to CAP station and has not arrived on station yet.

$$Tl_{kIA} = \begin{cases} \frac{t_a + t_{ID} + t1_{intc} + t_{cmb}}{p_{intc} \cdot p_{kIA}}, & \text{if } 0 < p_{intc} < 1 \text{ and } 0 < p_{kIA} < 1 \\ 0, & \text{if } p_{kIA} = 0 \text{ or } p_{intc} = 0 \end{cases} \quad (4.9)$$

and

$$T2_{kIA} = \begin{cases} \frac{t_a + t_{ID} + t2_{intc} + t_{cmb}}{p_{intc} \cdot p_{kIA}}, & \text{if } 0 < p_{intc} < 1 \text{ and } 0 < p_{kIA} < 1 \\ 0, & \text{if } p_{kIA} = 0 \text{ or } p_{intc} = 0 \end{cases} \quad (4.10)$$

The next issue to address is the way the interceptors are attritioned by the attackers. Due to the assumption that the attackers exhibit lethal self defense capability, define p_{kAI} as the probability that an attacker kills an interceptor during the air-to-air combat engagement. The value of p_{kAI} is determined based on the decision maker judgement, considering the attacker aircraft characteristics, Intelligence reports analysis, interceptor air-crew training level, etc. Let Tl_{kAI} be the expected time between two consecutive interceptors casualties when the attacker is engaged from CAP station; and Tl_{kAI} be the analogous variable for the case in which the attacker is engaged by a transient interceptor. The expression for these expected values are

$$T1_{kAI} = \begin{cases} \frac{t_a + t_{ID} + t1_{intc} + t_{cmb}}{p_{intc} \cdot p_{kAI}}, & \text{if } T1_{kIA} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.11)$$

and

$$T2_{kAI} = \begin{cases} \frac{t_a + t_{ID} + t2_{intc} + t_{cmb}}{p_{intc} \cdot p_{kAI}}, & \text{if } T2_{kIA} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.12)$$

The argument for these expressions is the same as for $T1_{kIA}$ and $T2_{kIA}$, because the number of intercept trial needed to kill one interceptor is also a random variable having a geometric distribution with parameter $p_{intc} \cdot p_{kAI}$. Notice that these expected values are conditional to the analogous interceptors' variables. This is so due to the fact that the attackers are not seeking to engage with the interceptors, so it is only possible for them to inflict any casualty to the interceptors if the interceptors do engage the attackers.

With the value for the expected times between casualties determined, we can define the attrition rate coefficients for the macro model for CAP management.

Let $\mu1_{IA}$ be the rate at which an individual interceptor from CAP station kills or neutralizes one arriving attacker; and $\mu2_{AI}$ be the rate at which an individual transient interceptor kills or neutralizes one arriving attacker. Let $\mu1_{AI}$ be the rate at which an arriving attacker kills or neutralizes an interceptor from CAP station;

and μ_{2AI} be the rate at which an arriving attacker kills or neutralizes a transient interceptor. The units for these variables are as follows

$$\mu_{iIA} \text{ is } \frac{\text{attackers killed}}{(\text{interceptors}) \cdot (\text{time})}, \quad i = 1, 2$$

$$\mu_{jAI} \text{ is } \frac{\text{interceptors killed}}{(\text{attackers}) \cdot (\text{time})}, \quad j = 1, 2$$

The expressions for each of these rates are

$$\mu_{1IA} = \begin{cases} \frac{1}{T1_{kIA}}, & \text{if } T1_{kIA} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.13a)$$

$$\mu_{2IA} = \begin{cases} \frac{1}{T2_{kIA}}, & \text{if } T2_{kIA} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.13b)$$

$$\mu_{1AI} = \begin{cases} \frac{1}{T1_{kAI}}, & \text{if } T1_{kAI} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.14a)$$

and

$$\mu_{2_{AI}} = \begin{cases} \frac{1}{T2_{kAI}}, & \text{if } T2_{kAI} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.14b)$$

and these are expressions to represent the attrition rates in the macro model for CAP management.

D. INDICATOR VARIABLES

The parameters studied on the previous section govern the *rate* at which aircraft leave each state of the system. However, in order for the system's behavior to make any physical sense, a flow between two states can occur at time t if and only if some specific condition holds at the state variable level at time t . Hence, to preserve the physical meaning of the system's behavior, a set of binary variables is used to detect pre-specified conditions whenever the flow between two interacting states takes place. Based on the pre-specified conditions, the binary variables turn *on* or *off* the flow between two interacting states. These binary variables are the indicator variables of the system, and are defined below.

$$I_{GND} = \begin{cases} 1, & \text{if } N_G(t) > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$I_{OC} = \begin{cases} 1, & \text{if } N_{OC}(t) > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$I_{CAP} = \begin{cases} 1, & \text{if } N_C(t) > 0 \\ 0, & \text{otherwise} \end{cases}$$

E. MODEL STRUCTURE

The definitions of system parameters, attrition rate coefficients and indicator variables carried out in the previous sections lay the grounds for the structure of the macro model for CAP management.

The state variables defined earlier in this chapter represent each of the aspects identified as germane for analysis purposes in an air defense scenario in which the interceptors are used in CAP. We have seen in the previous sections how the system parameters and the attrition rate coefficients represent the rate at which the interceptor aircraft represented in the model flow from one state to another within the system as time passes. The way this flow is regulated by the indicator variables were also addressed. These parameters and variables will now be used as building blocks of a system of the deterministic state transition equations that represents the scenario's interactions.

The changes with time in an air defense scenario using interceptors in CAP station can be represented by a set of Ordinary Differential Equations (ODE), provided the initial conditions at some time t_0 are known. Considering that such initial conditions are the basic set of information upon which the air defense planning work is made, a macro model for such a scenario can be modeled deterministically by the following system of differential equations:

$$\frac{dA(t)}{dt} = -\mu_{1AI} \cdot C1(t) - \mu_{2AI} \cdot C2(t) \quad (4.15a)$$

$$\frac{dN_G(t)}{dt} = \lambda_R - \lambda_{AI} \cdot [\bar{c} - N_C(t)] \cdot I_{GND} \quad (4.15b)$$

$$\frac{dN_{OC}(t)}{dt} = \lambda_{AI} \cdot [\bar{c} - N_C(t)] \cdot I_{GND} - \lambda_{oc} \cdot N_{OC}(t) - \lambda_s \cdot (1 - I_{CAP}) \cdot I_{OC} \quad (4.15c)$$

$$\frac{dN_C(t)}{dt} = \lambda_{OC} \cdot N_{OC}(t) - \lambda_{cb} \cdot N_C(t) - \lambda_s \cdot I_{CAP} \quad (4.15d)$$

$$\frac{dC1(t)}{dt} = \lambda_s \cdot I_{CAP} - C1(t) \cdot (\mu_{1IA} + \mu_{1AI}) \quad (4.15e)$$

$$\frac{dC2(t)}{dt} = \lambda_s \cdot (1 - I_{CAP}) \cdot I_{OC} - C2(t) \cdot (\mu_{2IA} + \mu_{2AI}) \quad (4.15f)$$

$$\frac{dN_B(t)}{dt} = \mu_{1IA} \cdot C1(t) + \mu_{2IA} \cdot C2(t) + \lambda_{cb} \cdot N_C(t) - N_B(t) \cdot (\lambda_{bc} + \lambda_{be}) \quad (4.14g)$$

$$\frac{dN_R(t)}{dt} = N_B(t) \cdot (\lambda_{bc} + \lambda_{be}) - \lambda_R \quad (4.15h)$$

$$\frac{dN_k(t)}{dt} = \mu_{1IA} \cdot C1(t) + \mu_{2IA} \cdot C2(t) \quad (4.15i)$$

with initial conditions:

$$N_G(0) = g;$$

$$N_{OC}(0) = oc;$$

$$N_C(0) = \bar{c};$$

$$C1(0) = c1;$$

$$C2(0) = c2;$$

$$N_B(0) = b;$$

$$N_R(0) = r;$$

$$N_K(0) = k; \text{ and}$$

$$A(0) = N_G(0) + N_{OC}(0) + N_C(0) + C1(0) + C2(0) + N_B(0) .$$

The argument from which the system of equations (4.15) is derived is as follows.

Equation (4.15a) represent the rate of change of $A(t)$ with time. Notice that $A(t)$ only decreases with time what makes intuitive sense because defense reinforcement is not being considered in the model. $A(t)$ decreases at a rate equivalent to the attackers attrition rate coefficients for each engagement; and this decrease is proportional to the number of interceptors engaged in each type of engagement.

Equation (4.15b) represents the rate of change of $N_G(t)$ with time. The first term on the right hand side represents the expected increase in $N_G(t)$ due to aircraft leaving the repair shop. The second term in the right hand side of (4.15b) represents the decrease in N_G with time. The factor $\lambda_{AJ} \cdot [\bar{c} - N_C(t)]$ is the demand for

interceptor on CAP station whenever the number of aircraft on station falls below the nominal force size to be kept on station; when such a demand occurs, it is attended by the air base at a rate λ_{A1} . But this will happen if there is some interceptor available on the ground at time t , what is controlled by the indicator variable I_{GND} .

Equation (4.15c) represents the rate of change of $N_{OC}(t)$ with time. The first term on the right hand side of this equation is the expected increase in $N_{OC}(t)$ caused by the interceptors who leave the air base going to CAP. Notice that this term is the same as the second term on the right hand side of equation (4.15b), what agrees with the intuition. The term $\lambda_{oc} \cdot N_{OC}(t)$ represents the decrease in the level of N_{OC} caused by those transient interceptors finishing the transit at a rate λ_{oc} . The third term, $\lambda_a \cdot (1 - I_{CAP}) \cdot I_{OC}$ represents the decrease in N_{OC} caused by those transient interceptors engaged in intercept during their way out to CAP. Note that this happens only when an attacker arrives in a moment in which the CAP station is empty, as indicated by the value of I_{CAP} ; also, interceptors can be engaged from their way out to CAP if there exist at least one transient interceptor at time t , what is controlled by the value of I_{OC} .

Equation (4.15d) represents the rate of change of $N_C(t)$ with time. The first term on its right hand side is the increase on the level of $N_C(t)$ caused by those transient interceptors who finishes their transit to CAP station, and is the same as the second term on the right hand side of equation (4.15c). The term $\lambda_{cb} \cdot N_C(t)$ represents the decrease in $N_C(t)$ caused by those interceptors who leave CAP station without engaging interception because no attacker arrives; this decrease occurs at a

rate λ_{cb} and is proportional to the number of interceptors on station at time t . The term $\lambda_a \cdot I_{CAP}$ is the decrease in $N_C(t)$ caused by those interceptors who leave the CAP to engage an arriving attacker; this decrease occurs at a rate λ_a , provided there is some interceptor on station, what is indicated by I_{CAP} .

Equation (4.15e) represents the rate of change in the level of $C1(t)$ with time. Its right hand side shows that $C1(t)$ is increased by those interceptors who have left CAP station to engage an arriving attacker, what happens at a rate λ_a , given that there is at least one interceptor on station. See that the first term of this right hand side is derived from the third term of the right hand side of equation (4.15d). The second term on the right hand side of equation (4.15e) is the expected decrease on the level of $C1(t)$ with time. Such decrease is proportional to the number of aircraft engaged in the fight, and is caused by the mutual attrition that occurs in the air-to-air combat engagement, what occurs at a rate $(\mu1_{IA} + \mu1_{AI})$.

Equation (4.15f) represents the rate of change of $C2(t)$ with time. The first term on its right hand side represents the expected increase in the level of $C2(t)$ and is the same as the last term on the right hand side of equation (4.15c). The rationale for the second term on this equation right hand side is analogous to that for the second term on the right hand side of equation (4.15e).

Equation (4.15g) represents the rate of change of $N_B(t)$ with time. $N_B(t)$ is fed by three distinct states as can be seen in Figure 4. The three first terms on the right hand side of this equation is the expected increase in the number of interceptors in the transit back to the air base with time. Observe that the first term represents those

air combat winner interceptors generated by the second term of equations (4.15e), and the second term represent those winner generated by the second term of equation (4.15f); to these returning interceptors it is added those who did not engage the fight and are flying back to the air base from the CAP station, what is represented by the expression $\lambda_{cb} \cdot N_C(t)$ in the third term; this expression is the same as in the second term of equation (4.15d). The expected decrease in the number of interceptors flying back to the air base with time is represented by the last term in this equation; such decrease is proportional to the number of returning interceptors at time t , and happens at a rate equals to $(\lambda_{be} + \lambda_{bc})$.

Equation (4.15h) represents the rate of change of $N_R(t)$ with time. Its right hand side shows that $N_R(t)$ is increased by the same amount by which $N_B(t)$ is decreased, as can be verified by the fact that the first term on this equation's right hand side is the same as the last term on the right hand side of equation (4.15f). The expected decrease in the level of $N_R(t)$ is equivalent to the increase in the level of $N_G(t)$ because the second term in this equation's right hand side is the same as the first term on the right hand side of equation (4.15b).

Equation (4.15i) represent the rate of change of $N_k(t)$ with time. On the contrary of $A(t)$, $N_k(t)$ only increases with time what also makes intuitive sense. Observing Figure 4 one will note that $N_k(t)$ is not represented there; it also can be seen that $N_k(t)$ plays no functional role in the system of equations. The reason this quantity is accounted for in the system is because it will be used in the Measures of Effectiveness (MOE) in the numerical examples presented in next Chapter.

V. NUMERICAL EXAMPLES

The macro model for CAP management described in the previous Chapter will be used as a tool to analyze the option of CAP stationing which renders the best results for overall the air defense mission.

Two distinct scenarios were used in the three numerical examples shown below. Both scenarios represent typical tactical environments where the defense faces EW limitations. In one of the scenarios solutions were obtained for two different intercept tactics: interceptors using subsonic air speed during the interception procedure, and interceptors using supersonic air speed during the interception procedure.

A. THE SCENARIO AND INITIAL CONDITIONS

The following conditions are common to both scenarios:

- (1) The attackers are equally likely to approach the target area within a sectors of 120 degrees. This sector is referred to as threat sector.
- (2) The angle formed between the bisector of the threat sector and the line segment connecting the air base location to the target location measures 135 degrees; the air base is at 43 nautical miles from the target position.
- (3) The interceptor aircraft is considered as being an F-5E type of aircraft which takes off from the air base armed with two side-winder infra-red air-to-air missiles and 560 rounds of 20 mm cannon air-to-air ammunition; and is equipped with an external 275 US gal expendable fuel tank.

(4) The hypothetical attacker aircraft is flying at 20000 feet, at an air speed equivalent to Mach Number 0.7, and features a limited air-to-air lethal self-defense capability, with flight characteristics yielding a sustained turn rate of 5.0 degrees per second when the encounter conditions are considered⁵

(5) The attacking air force is capable of maintaining a constant effort of one raid at each 15 minutes interval for a 12 hours period.

(6) It is considered that, in the average, it takes ten sustained turns to the interceptor air crew to obtain firing position during an air-to-air combat engagement, when using the interceptor aircraft and engaging against an aircraft similar to one used by the attacker air force.

(7) The probability of success in each intercept trial is considered to be 0.97.

(8) It is considered that in each air-to-air combat engagement the interceptor aircraft has a probability of killing the attacker equal to 0.75, and the probability that the interceptor is killed by the attacker is 0.1.

(9) The initial number of interceptor aircraft available in the defense air force inventory is 20.

(10) The identification time of the C³ system, t_{ID} , is one minute, counted from the moment the attacker is first detected by the early warning radar system (EWR).

(11) The scramble time in the air base, t_{AB} , is two minutes.

(12) All interceptor aircraft have to undergo maintenance service between two consecutive sorties.

(13) There are ten repair crew in the air base maintenance shop. This number is kept constant during the air campaign considered.

(14) The time it takes to one repair crew to service one interceptor aircraft in the maintenance shop, t_{rep} , is 30 minutes.

In one scenario, which we will call scenario 1, the early warning average distance is considered to be 60 nautical miles, and the anti-air weapons effective

⁵ For actual planning these value is obtainable from the attacker aircraft Flight Manual analysis.

range is ten nautical miles. In the other scenario, called scenario 2, it is considered an average early warning range of 150 nautical miles, the same anti-air weapons capabilities as in scenario 1, but the attackers are assumed to be using stand-off weapons with a release range of 40 nautical miles from the target area. Hence, in scenario 1 the attackers have to be destroyed/neutralized by the interceptors at or before they reach a 10 nautical miles range from the target area, whereas in scenario 2 this have to be accomplished at 40 nautical miles from the target area.

B. MEASURES OF EFFECTIVENESS (MOE)

To assess the performance of the interceptors in CAP as a defensive weapon system the following quantities were used:

- (1) MOE1: Number of attackers casualties due to CAP interceptors - this quantity gives a measure of the lethality of the CAP interceptors. It can be used in further analysis (not carried out in this study) to assess the contribution of the CAP to the overall effectiveness of the integrated air defense system (IADS). This quantity should be maximized.
- (2) MOE2: Number of CAP interceptors casualties due to engagements with the attackers - this quantity measures the capability of the CAP interceptors in interacting with the attackers lethality. It can be used in further analysis to assess how the CAP option impacts the interceptors survivability. This quantity should be minimized.
- (3) MOE3: Kill ratio - this quantity measures the ratio between the attackers casualties and the CAP interceptors casualties. It gives a measure, from the attacking air force perspective, of the cost in attacker aircraft the intruders have to pay due to the CAP interceptors. This quantity should be maximized.
- (4) MOE4: Number of attackers surviving the CAP interceptors - this quantity gives a measure of the "leakage" of attackers aircraft that will have to be engaged by the anti-air weapons system. Because the capabilities of the anti-air weapons system are usually measured by the number of targets that can be engaged at a time, MOE4 will be expressed as the rate at which the attackers reach the anti-air

weapons effective range line, i.e., attackers per minute. This quantity should be minimized.

C. PARAMETERS COMPUTATIONS

To assess the CAP performance in scenario 1, the CAP was stationed at 10, 30 and 40 nautical miles from the target area. In scenario 2 it was considered the CAP stationed at 40, 60, 80, 100, 120, and 140 nautical miles from the target area. As it was seen in Chapter IV, the numerical values of many of the parameters depends on the CAP distance from the target area because they are functions of the time variables involved, and to evaluate these time variables the flight characteristics of the interceptor aircraft have to be considered. Because the interceptor aircraft is assumed to be an F-5E type of aircraft, the computations below are based on data found in the NORTHROP F-5E Flight Manual [Ref. 12]. In what follows all distances are measured in nautical miles (N.M.); the fuel quantities are measured in pounds of fuel (lb); the time is measured in minutes unless otherwise stated; the altitude is expressed in feet (ft); and the air speed is measured either in Mach Number (M), or in nautical miles per minute (N.M./min).

Force Size on CAP (\bar{c}): using the probabilities considered above and equation (4.5) the value of \bar{c} for both scenarios is obtained as follows:

$$\bar{c} = \lceil \left[\frac{1}{p_k^* \cdot p_{int}^*} \right] \rceil = \lceil \left[\frac{1}{0.75 \cdot 0.97} \right] \rceil = \lceil \left[\frac{1}{0.7275} \right] \rceil = 2 \text{ interceptors}$$

CAP altitude selection: considering the attacker aircraft characteristics and the interceptor take off configuration given above, the CAP altitude selection should give the interceptor an altitude advantage over the expected threat because the interceptor's short-range weapons characteristics. Furthermore, it should be such that the interceptor's endurance during the loiter phase is improved. According to the aircraft's Flight Manual the loitering altitude for maximum endurance fuel consumption setting varies with the initial aircraft gross weight [Ref. 12:p. A5-3]. The initial altitude recommended is 27000 ft, but to improve the interceptors chances to acquire the attacker by visual means, the CAP altitude adopted will be 25000 ft.

Time and fuel to fly out to CAP station (t_{oc}) and (f_{oc}): using data from the performance charts in the aircraft's Flight Manual the following expressions were derived for these quantities as a function of the CAP station distance from the air base [Ref. 12:pp. A4-17, A4-18]:

$$t_{oc} = \begin{cases} 6.5 \text{ min}, & \text{if } DBC \leq 53 \text{ N.M.} \\ (0.125 \cdot DBC - 0.125) \text{ min}, & \text{if } DBC > 53 \text{ N.M.} \end{cases}$$

$$f_{oc} = \begin{cases} 670 \text{ lb}, & \text{if } DBC \leq 53 \text{ N.M.} \\ (6.25 \cdot DBC - 339) \text{ lb}, & \text{if } DBC > 53 \text{ N.M.} \end{cases}$$

where DBC is the distance from air base to CAP station.

Maximum time the interceptor can stay engaged in air-to-air combat (t_{cmb}^*) and the fuel for combat (f_{cmb}^*): using performance data from the Flight Manual the following expressions were derived for f_{cmb} and t_{cmb} , respectively [Ref 13:p. A8-11]:

$$f_{cmb} = (200 \cdot t_{cmb}) \text{ lb, and } t_{cmb} = (0.005 \cdot f_{cmb}) \text{ min}$$

To compute t_{cmb}^* we use the fact that, under the conditions listed above, the interceptor aircraft will have a sustained turn rate in the order of 7.5 degrees per second [Ref. 12:pp. A8-46, A8-48]. In the present examples it is assumed that the attacker's turn characteristic is known. If this is not the case, one can derive the attacker's turning performances, provided some specific data about the attacker aircraft is available, by using the Energy-State Approximation theory as found in Nicolai [Ref. 6:pp. 3-21-3 - 3-28]. Knowing the turn rate of the two aircraft, we see that the interceptor has a turn rate margin advantage of 2.5 degrees per second. Hence, using the same rationale as described in Chapter III, Section D, Subsection 1, we divide 180 by this margin advantage and find that the minimum time an air-to-air combat engagement between these two aircraft can theoretically last is 72 seconds. In the scenario's conditions it is given that, in the average, it takes ten sustained turns to the interceptor's air crew to obtain firing position in air-to-air combat engagement when engaging a similar aircraft, so this information is used here. A 180 degrees sustained turn with the interceptor aircraft takes (180 degrees + 7.5 degrees per second) = 24 seconds, implying that ten of such turns will last 240 seconds; summing these two quantities and considering one sustained turn more for disengagement we get 336 seconds of combat duration. Form these figures it seems

reasonable to consider a t_{cmb}^* equals to 5.5 min, and once we having the value of t_{cmb} , to obtain the value of f_{cmb}^* is straightforward. These values of t_{cmb}^* and f_{cmb}^* are the ones that give the interceptor a probability of kill equal to 0.75.

The selection of the interceptor/attacker airspeed relationship (K): usually the value of K is selected such that the interceptor air speed, in addition of making the intercept geometrically feasible, also gives the interceptor the conditions to engage the air-to-air combat at the best of its maneuverability characteristics yet preserving fuel. For the sake of example we are considering in one of the scenarios the use of supersonic intercept air speed. For the interceptor aircraft being used in the present examples the most favorable subsonic air speed for interception is Mach Number 0.92, what gives a value of K equals to 1.31. For the supersonic scenario it will be considered an air speed equivalent to Mach Number 1.2, giving a K equals to 1.71.

The mean time to intercept (t_{intc}) and the fuel for interception (f_{intc}): to compute these quantities, it must be considered that the attackers are equally likely to approach the target area from any direction within the threat sector. So it is reasonable to assume that the CAP will be stationed along the threat sector bisector line. To compute the *average* values of t_{intc} and f_{intc} to be used in the macro model for CAP management, one must first to determine the *average* value of d_{intc} , the distance covered by the interceptor during the intercept procedure. As seen in Chapter IV, Section C, Subsection 1, the values of θ which d_{intc} is maximum and minimum in a given scenario is easily determined. For each of these values of θ , the quantity S, i.e., the intercept range as measured from the target position, is computed

using equation (3.8), because in both scenarios we have $K \neq 1$. Having the largest and the smallest possible values of S in the scenario, equation (3.6) gives the corresponding values of P , the attacker's range at the beginning of the intercept, as measured from the target position. And once the values of P are known, the respective values of d_{intc} are determined using the fact that, according to Figure 2, $d_{intc} = K \cdot (P - S)$. The *average* value of d_{intc} in each scenario is the mean value of largest and the smallest d_{intc} found in this process. The above computations to find the average value of d_{intc} have to be carried out for each scenario, for each different value of K , and for each CAP station position selected. Considering the two values of K used in these numerical examples, the following expressions for f_{intc} and t_{intc} as functions of d_{intc} were derived from the interceptor's Flight Manual:

when $K = 1.31$ [Ref. 12:pp. A8-11, A8-18],

$$f_{intc} = \begin{cases} (20.45 \cdot d_{intc}) \text{ lb,} & \text{if } d_{intc} \leq 22 \text{ N.M.} \\ (10.45 \cdot d_{intc} - 220) \text{ lb,} & \text{if } d_{intc} > 22 \text{ N.M.} \end{cases}$$

$$t_{intc} = \begin{cases} (0.114 \cdot d_{intc}) \text{ min,} & \text{if } d_{intc} \leq 22 \text{ N.M.} \\ (0.109 \cdot d_{intc} + 2.5) \text{ min,} & \text{if } d_{intc} > 22 \text{ N.M.} \end{cases}$$

when $K = 1.71$ [Ref. 12:pp. A8-11, A8-14],

$$f_{intc} = \begin{cases} (20 \cdot d_{intc}) \text{ lb,} & \text{if } d_{intc} \leq 25 \text{ N.M.} \\ (125 \cdot d_{intc} - 730) \text{ lb,} & \text{if } d_{intc} > 25 \text{ N.M.} \end{cases}$$

$$t_{intc} = \begin{cases} (0.1 \cdot d_{intc}) \text{ min,} & \text{if } d_{intc} \leq 25 \text{ N.M.} \\ (0.0833 \cdot d_{intc} + 0.416) \text{ min,} & \text{if } d_{intc} > 25 \text{ N.M.} \end{cases}$$

The mean values of the recovery time (t_{be}) and the fuel to recovery (f_{be}): to compute these two quantities the mean recovery distance, d_{rec} have to be determined first. The largest and the smallest values possible of d_{rec} are determined for each scenario and for each value of K using the same procedure as described in Chapter IV, Section C, Subsection 1. With the mean value of d_{rec} determined, the values for t_{be} and for f_{be} are computed using the following expressions derived from the Flight Manual performance charts [Ref. 12:p. A4-25]:

$$t_{be} = \frac{d_{rec}}{8.0} \text{ min} \quad \text{and} \quad f_{be} = (6.1 \cdot d_{rec} + 53.44) \text{ lb}$$

The maximum time on station (t_m): with the mean values of fuel consumption for each phase of the CAP interceptor mission in each scenario computed above, it is possible to compute the maximum time an interceptor can stay loitering on station if no attacker arrives, t_m . To this end, equation (4.3) is used to determine f_{CAP} , the amount of fuel apportioned for loitering on station. Having this quantity, the following expression is used to compute the value of t_m in the given scenarios, for the CAP altitude considered [Ref. 12:p. A5-3]:

$$t_m = (0.023 \cdot f_{CAP} - 4.23) \text{ min}$$

Once the mean values of the time variables for the scenarios are computed, the parameter computations are straightforward following the parameter definitions given in Chapter IV, Section C, Subsection 2. For each CAP distance selected, the actual value of t_{cmb} is computed considering the minimum value of the intercept range S found previously. If the intercept range is less than the quantity $h + t_{cmb}^* \cdot v_a$, the value of t_{cmb} is calculated according to equation (4.2). This actual value of t_{cmb} is used then to adjust the probability of kill according to equation (4.7), and the attrition rates are finally calculated as prescribed by equations (4.13) and equations (4.14).

For each of the three different situations given by the two scenarios above the initial values used for each state variable of the macro model for CAP management was as following:

$A(0) = 20$, meaning that the initial defense inventory is 20 interceptors, and no interceptor was lost at time 0;

$N_G(0) = 14$, meaning that the defense starts with 14 interceptors ready to fly in the air base;

$N_{OC}(0) = 0$, meaning that there is no transient interceptor at time 0;

$N_C(0) = 2$, meaning that the defense starts with 2 interceptors on station;

$C1(0) = 0$, meaning that there is no CAP interceptor engaged at time 0;

$C2(0) = 0$, meaning that there is no transient interceptor engaged at time 0;

$N_B(0) = 0$, meaning that there is no interceptor flying back to the air base at time 0;

$N_R(0) = 4$, meaning that the defense starts with 20% of its inventory under repair;

$N_k(0) = 0$, meaning that no attacker was killed at time 0.

D. NUMERICAL SOLUTIONS

A computer program using FORTRAN 77 was developed to compute the parameters according to the procedures described above. It worth emphasizing that such a program checks for intercept feasibility at each time variable or fuel variable computation, considering the CAP stationing being used.

The system of equations (4.15) was solved by means of the software package MATLAB which uses the automatic step size Runge-Kutta-Fehlberg integration method [Ref. 13:p. 3-139]. Considering the MOE's adopted, it was assumed that the attacker air force has unlimited supply of airplanes. With this assumption, each scenario condition was ran *simulating* a 12 hours period with a constant arriving rate, in order to be possible to detect any trend in the results. A time history plot of the results of each scenario is shown and analyzed in the next Section.

E. NUMERICAL RESULTS AND ANALYSIS

The results of the model runs are presented in the plots below. The plots are grouped by MOE, and each plot contains the results of all CAP stationing options for the scenario considered.

1. Scenario 1

In Figures 7, 8, 9, and 10 below we have the results for each MOE regarding the CAP stationing options in scenario 1. As far as MOE1 is concerned, the inspection of Figure 7 shows that option for stationing the CAP at 10 N.M. range from the target area is the least recommended. It is clear that stationing the CAP further away from the target will cause more damage to the attackers. Placing the CAP either at 30 N.M. or 50 N.M. has equivalent performances in the beginning of the battle, up to around the third hour of battle; from this point in time on, the performance of the CAP at 50 N.M. range dominates markedly.

Besides the fact that it gives a clear idea of the damage caused to the attackers, MOE1 does not show how much these results cost to the defense. One way of obtaining this type of assessment is by means of MOE2 in Figure 8 below. This Figure shows an equivalent performance of the three options in the very beginning of the period, with some noticeable difference appearing again around the third hour of battle. Considering the fact that the quantity of MOE2 should be minimized, the inspection of Figure 8 shows that the best option in this regard is to place the CAP at 30 N.M. from the target area.

However, if we observe the magnitude of the values on the y-axis scale, it can be seen that the difference has any physical meaning only by the end of the 12 hour period.

The analysis of MOE3 in Figure 9 gives a better mean of comparison among the relative performance of each option. It shows how the CAP stationing can affect

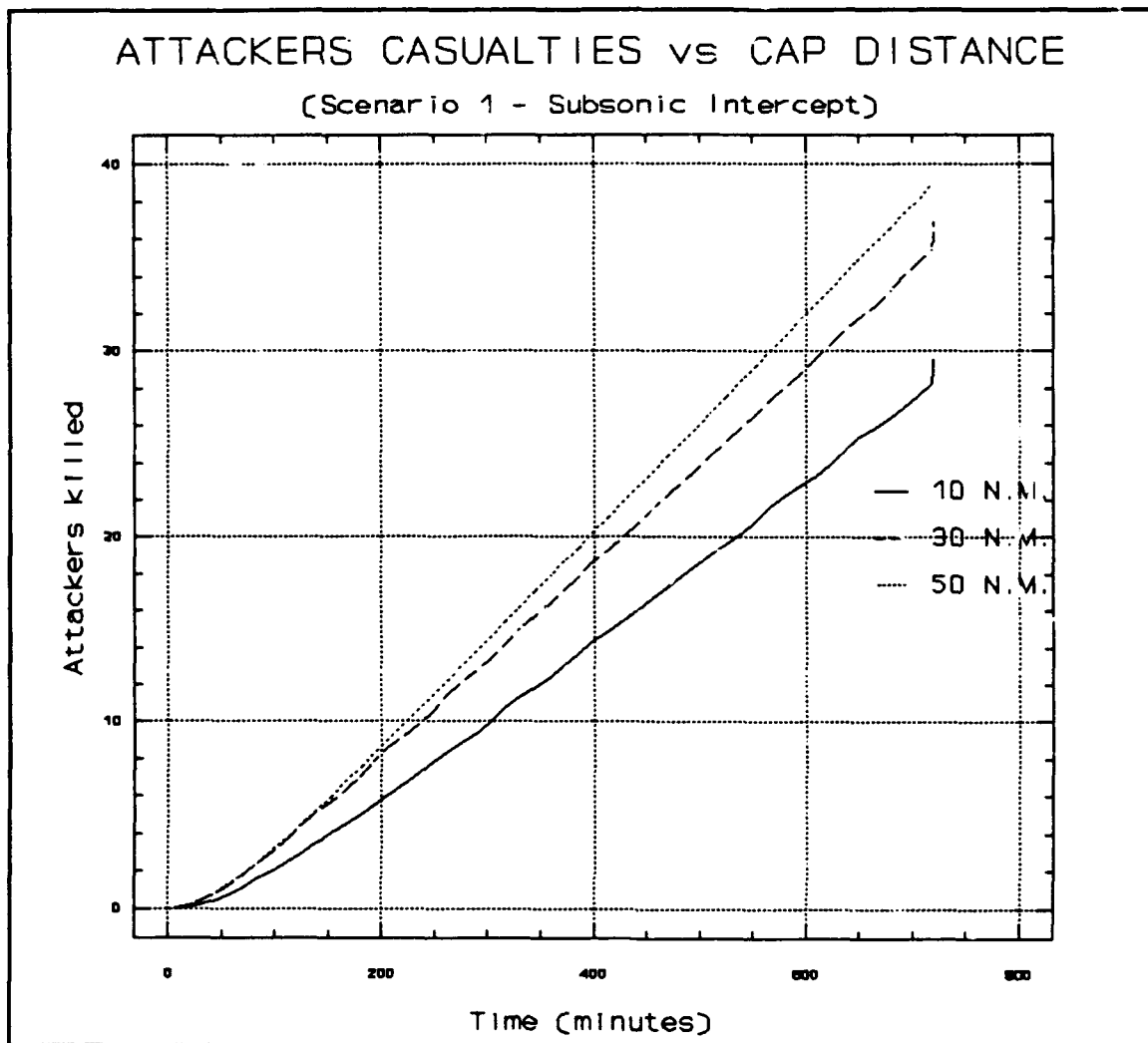


Figure 7. Scenario 1, MOE1: Attackers Casualties

the interceptor's effectiveness in terms of the cost of attacker aircraft per interceptor lost in the battle. The option which maximizes this quantity is the CAP at the 30 N.M. range.

This Figure shows that stationing the CAP at 30 N.M. makes the CAP interceptors almost one unit better than having them loitering at the 50 N.M. range, and more than three units better than when they are at the 10 N.M. range.

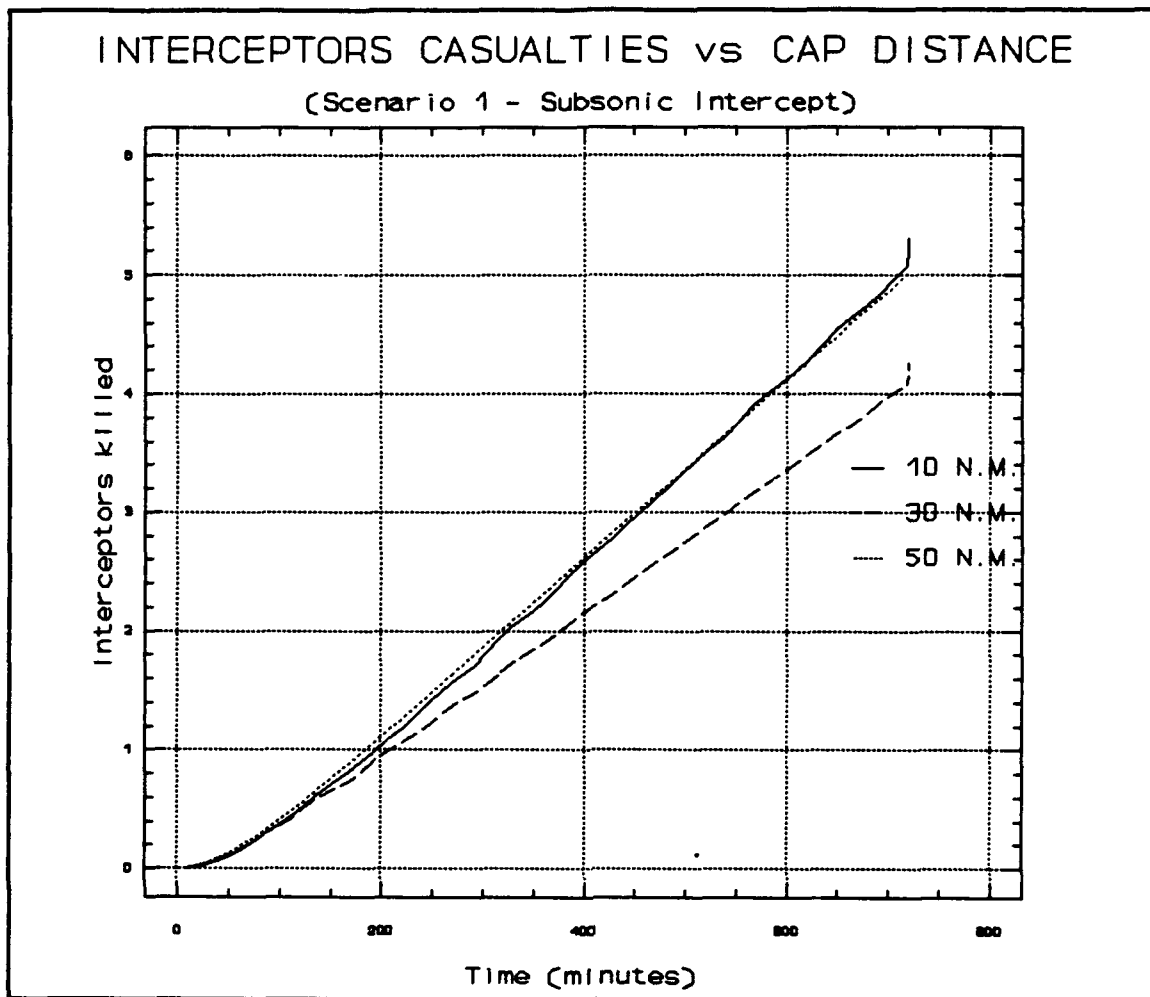


Figure 8. Scenario 1, MOE2: Interceptors Casualties

MOE4 is shown in Figure 10 below. It gives a measure of how much the CAP interceptors alleviate the anti-air weapons system's work load. The option for the CAP at 30 N.M. is again the one which yields the best results considering that such an effective measure should be minimized.

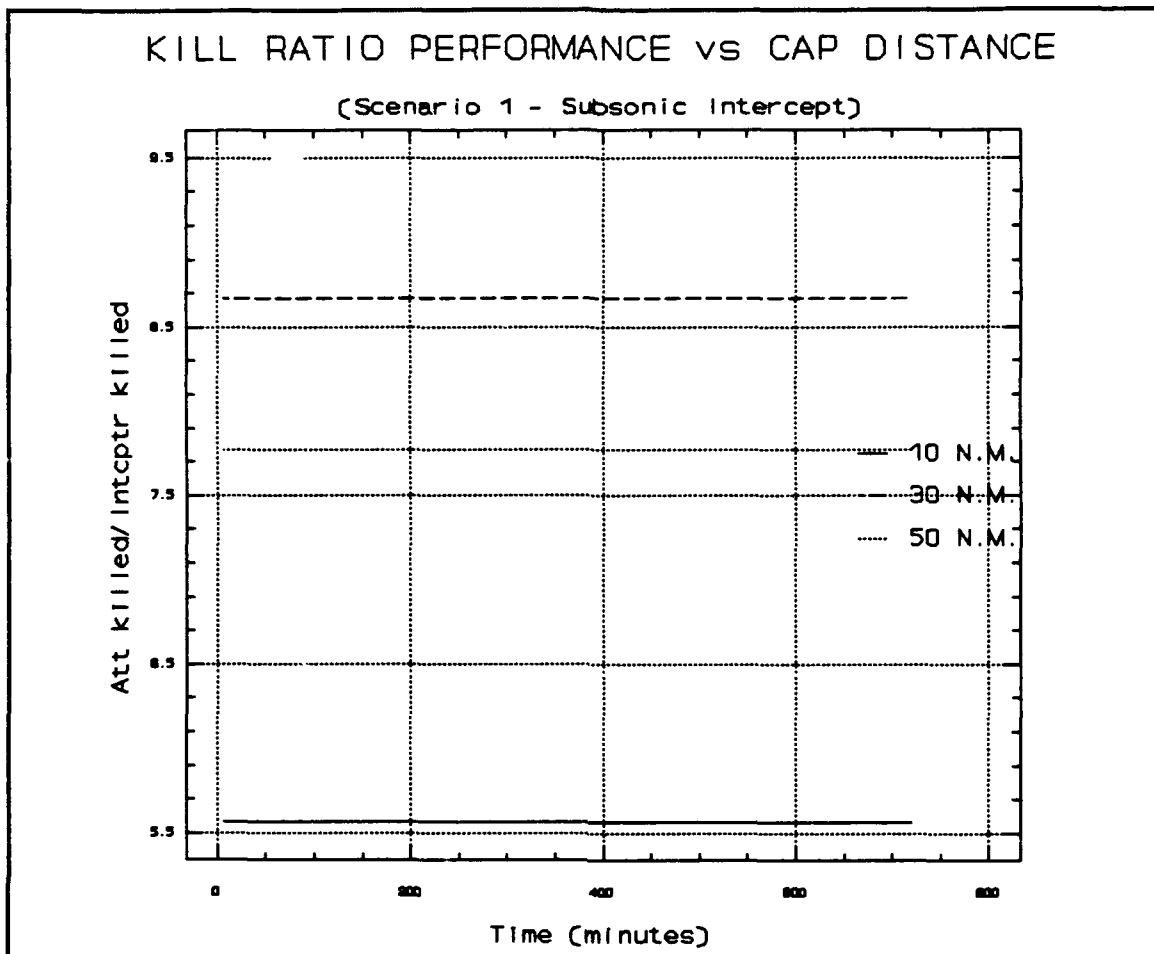


Figure 9. Scenario 1, MOE3: Kill Ratio

2. Scenario 2

The subsonic intercept in scenario 2 is shown in Figure 11. It is observed that in the first hour of activities all options perform fairly alike. From this point in time on, a definite distinction among the options is noticed.

The 140 N.M. option dominates the performance ranking through out the time period analyzed, reaching the total of 42 attackers killed. It is also observed that the other options present the same general behavior as that of the best option, the least effective of them all getting at the end of the period with around 30 enemies killed.

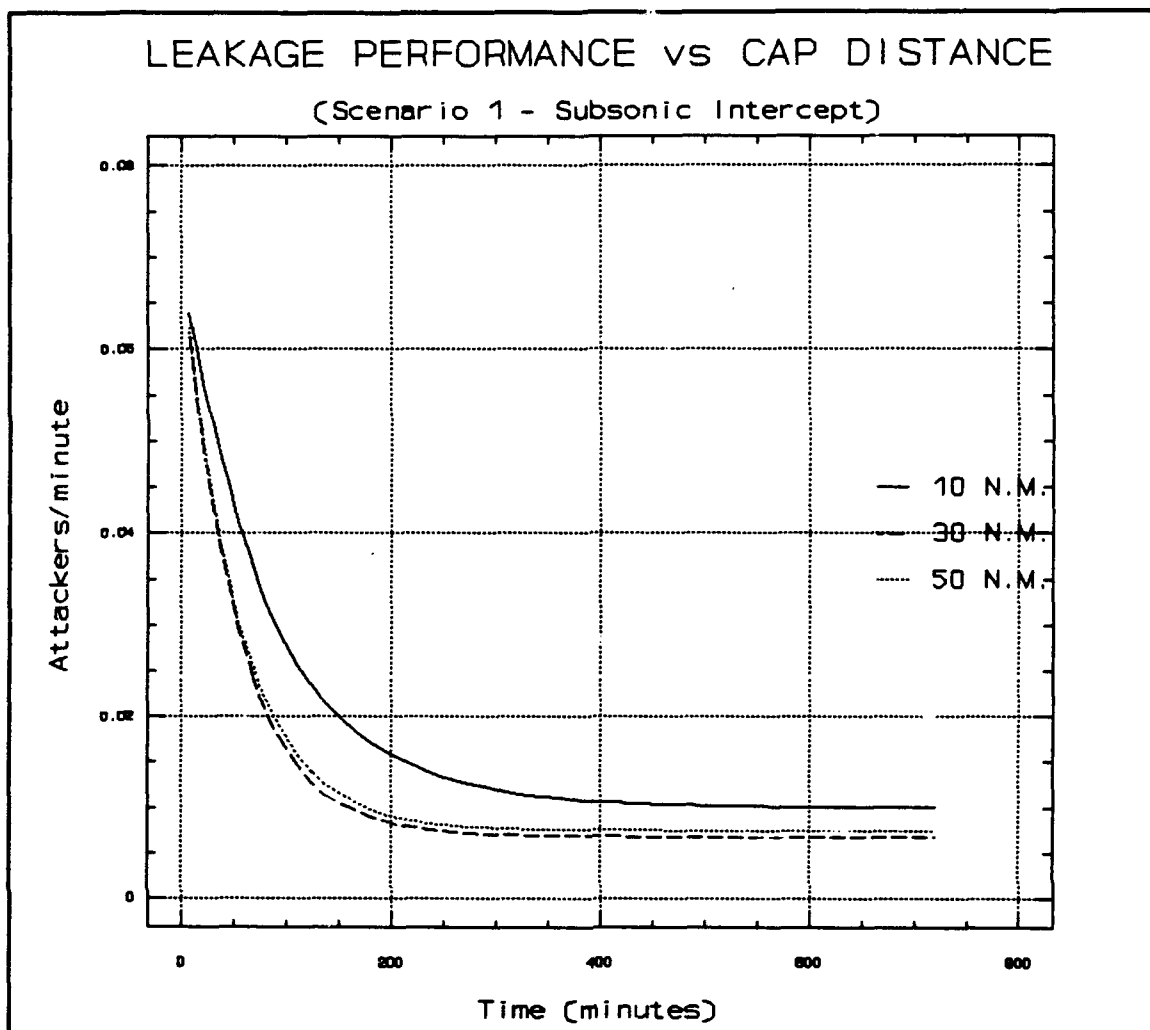


Figure 10. Scenario 1 MOE4: CAP Leakage

When Figure 12 is analyzed a different result is observed. Options for stationing the CAP either at 80 N.M. or at 100 N.M. presents a general dominance through out the period, reaching the score of about 40 kills. Nevertheless, the most significant aspect in this Figure is the behavior of the two most distant CAP options. Notice that their killing scores remain well below those of other options. In Figure 13 there is a subplot of Figure 12 focusing on the initial phase of the period. It can be seen that during this phase the CAP interceptors attrites the attackers at a very

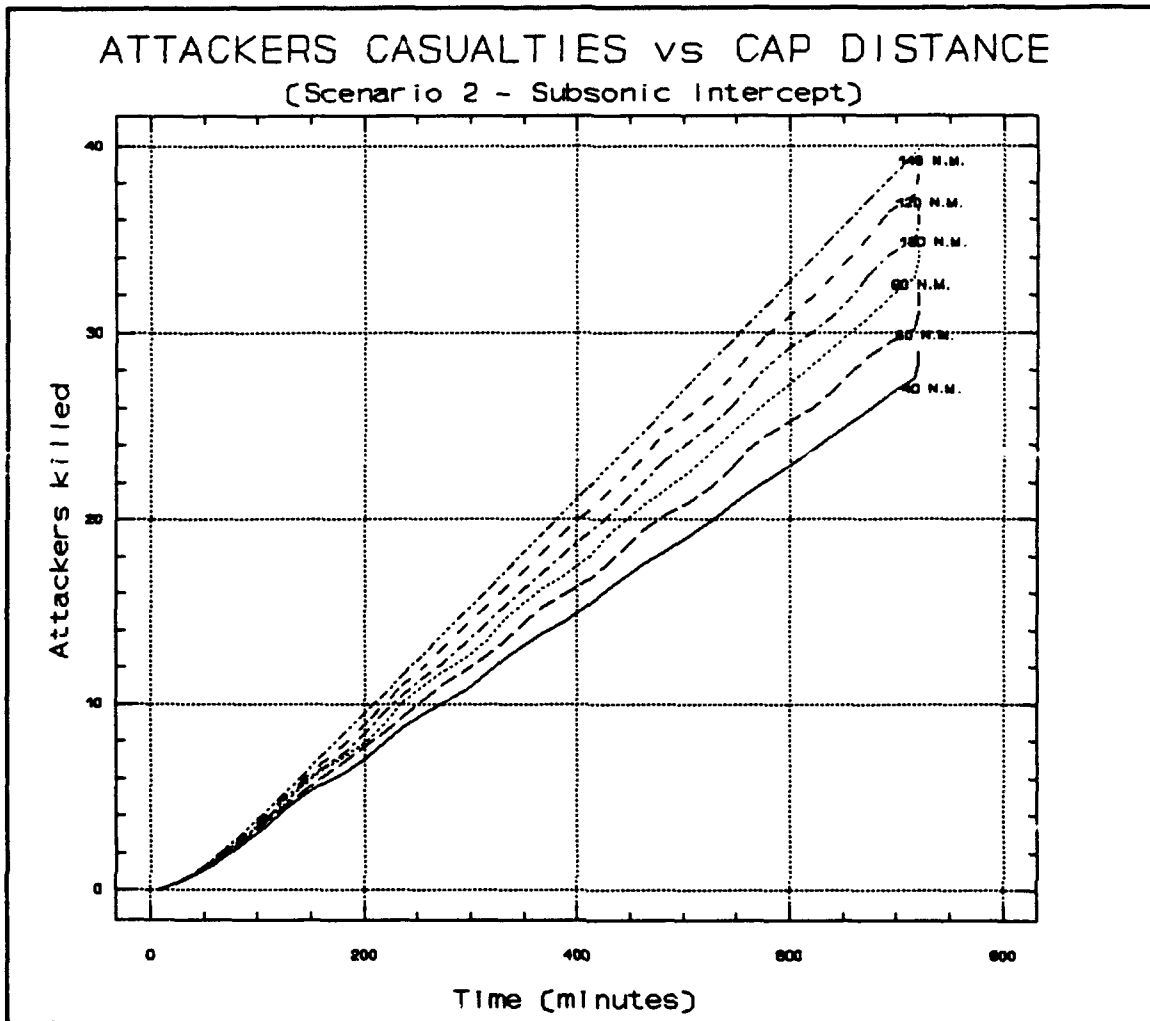


Figure 11. Scenario 2 (subsonic), MOE1: Attackers Casualties

slow pace if compared with the other options. Notice also that during the initial phase of activities the 100 N.M. is not dominant.

In Figure 14 we have the results for MOE2 from the subsonic intercept case in scenario 2. Considering that the quantity regarding this MOE should be minimized, the options perform in the inverse order as they did for MOE1. Notice that after a 12 hours battle the largest difference between options is of barely one aircraft. See that with subsonic intercept there will have fuel enough for the

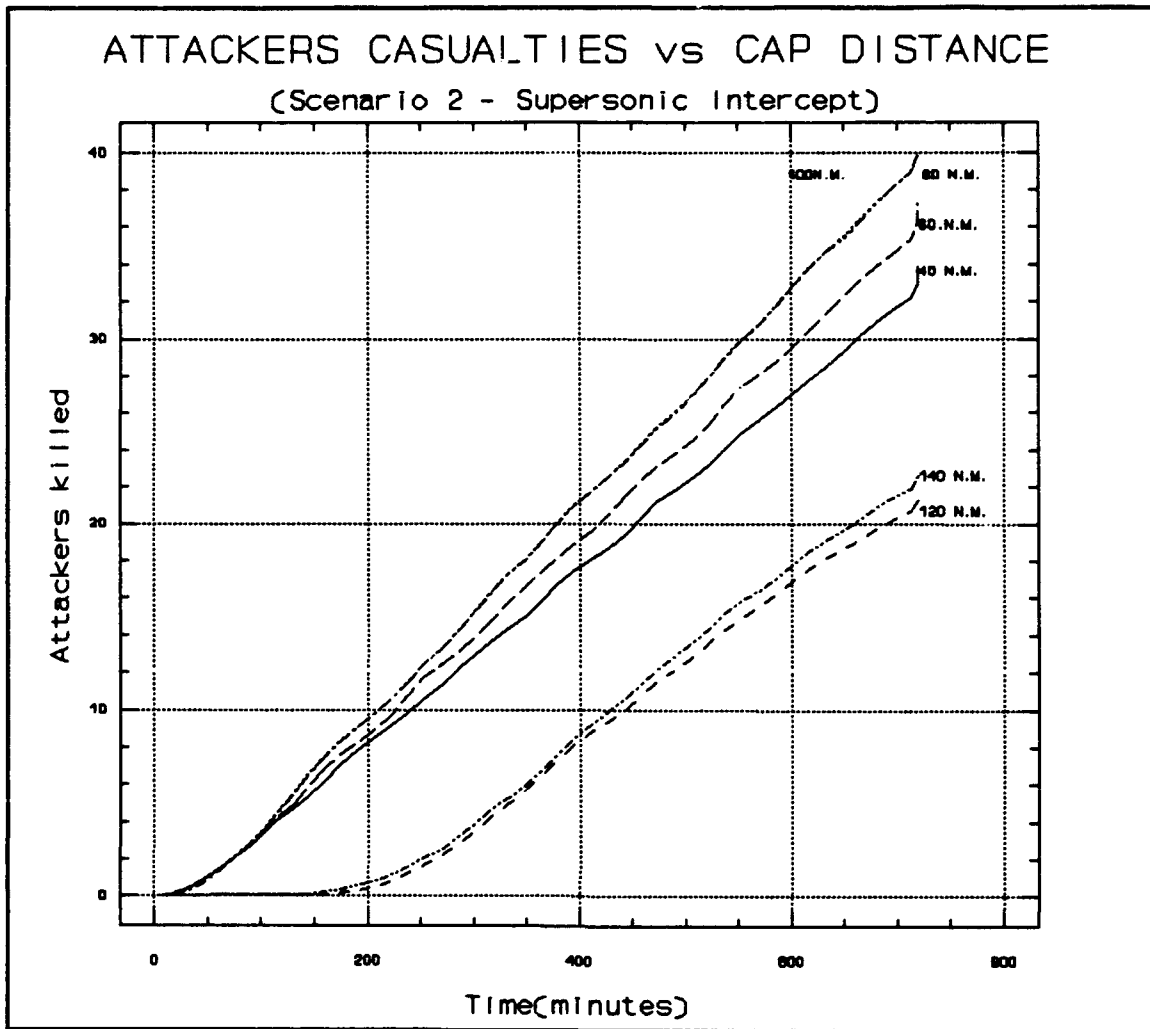


Figure 12. Scenario 2 (supersonic), MOE1: Attackers Casualties

interceptor to engage the air-to-air combat in all CAP stationing options because in this case there is no correction in the expected probability of kill. Hence, all options yield similar performances.

The plot of the results regarding MOE2 for the supersonic case in scenario 2 is shown in Figure 15, and in Figure 16 there is a subplot of Figure 15.

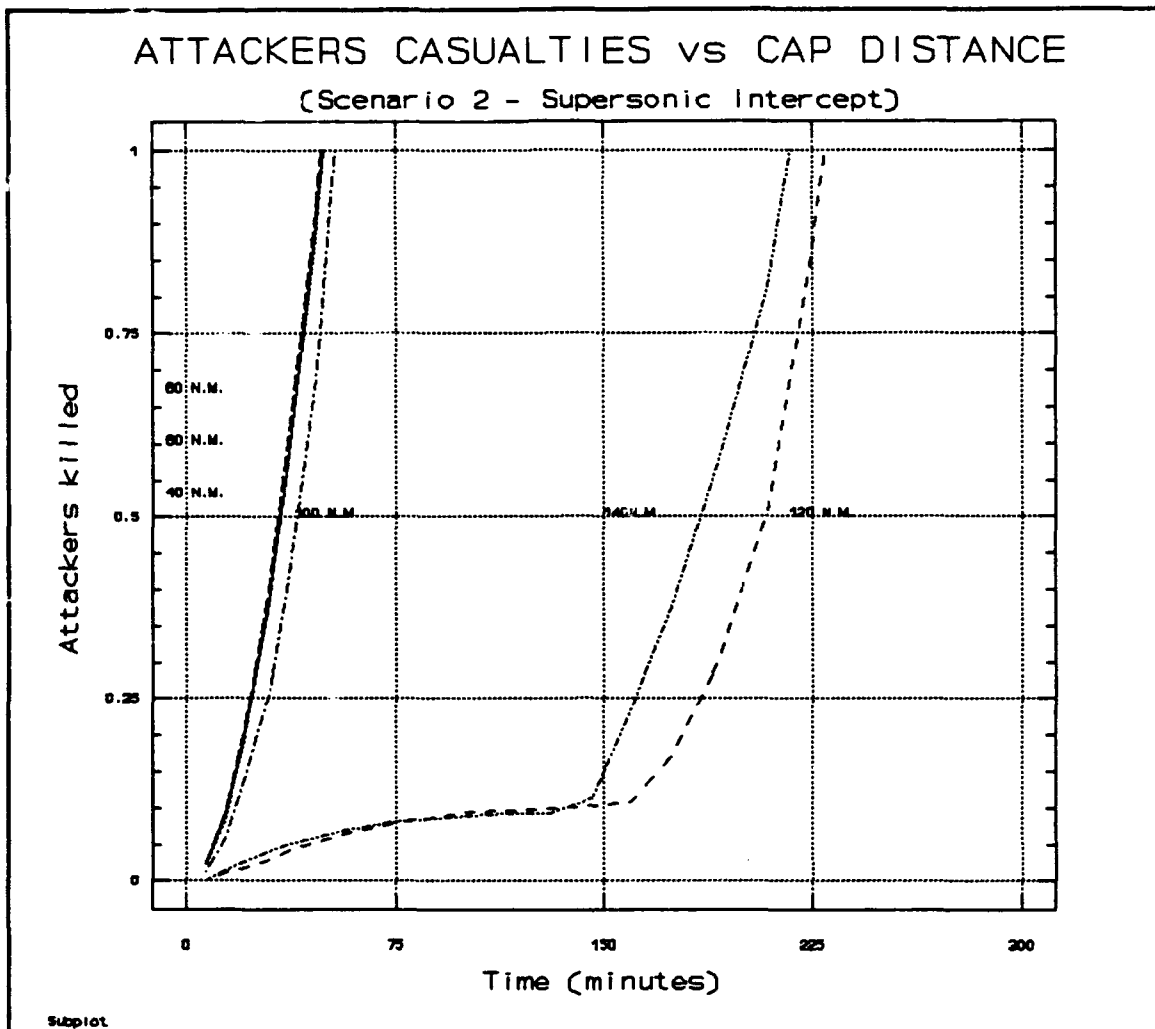


Figure 13. Subplot from Figure 12

The same behavior is observed as for MOE1. Now the least damage to the interceptors occurs when the CAP is stationed further away from the target. Comparing this plot with the one in Figure 12, it is seen that the two further CAP options are attrited at a pace analogous at that at which they impose to the attackers. One explanation for this fact is found in the fuel consumption analysis presented in Chapter III, Section D, Subsection 2. In the present case it applies as follows.

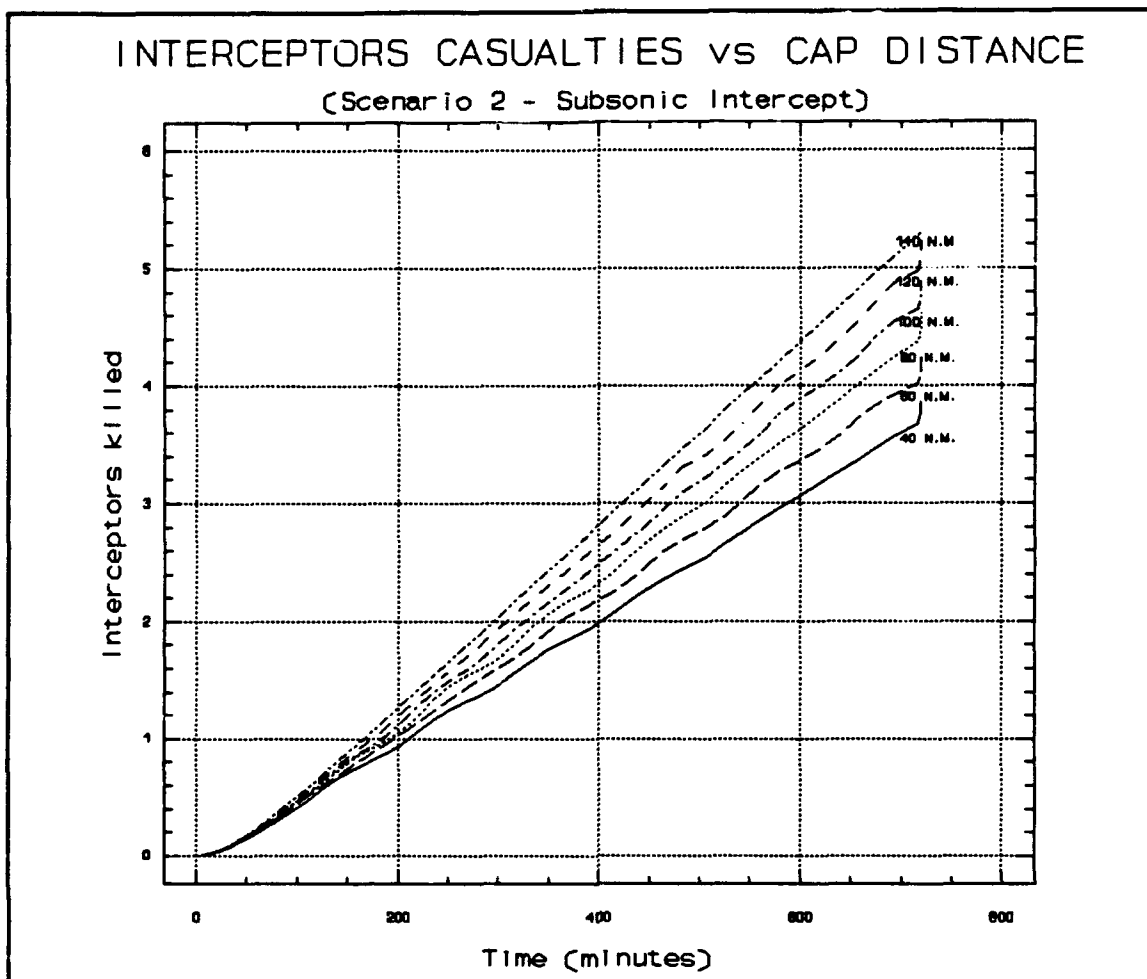


Figure 14. Scenario 2 (subsonic), MOE2: Interceptors Casualties

To station the CAP at longer distance from the target area in such a scenario implies that it will be placed it at longer distance from the air base also. This presupposes a greater amount of fuel being apportioned for the transit out phase of the mission. On the other hand, a further away CAP also signifies that the intercept will occur at further distances from the air base, hence increasing the fuel needed for recovery. In addition to these fuel demand increases, the high air speed setting during the interception will demand another great amount of fuel being allotted to the

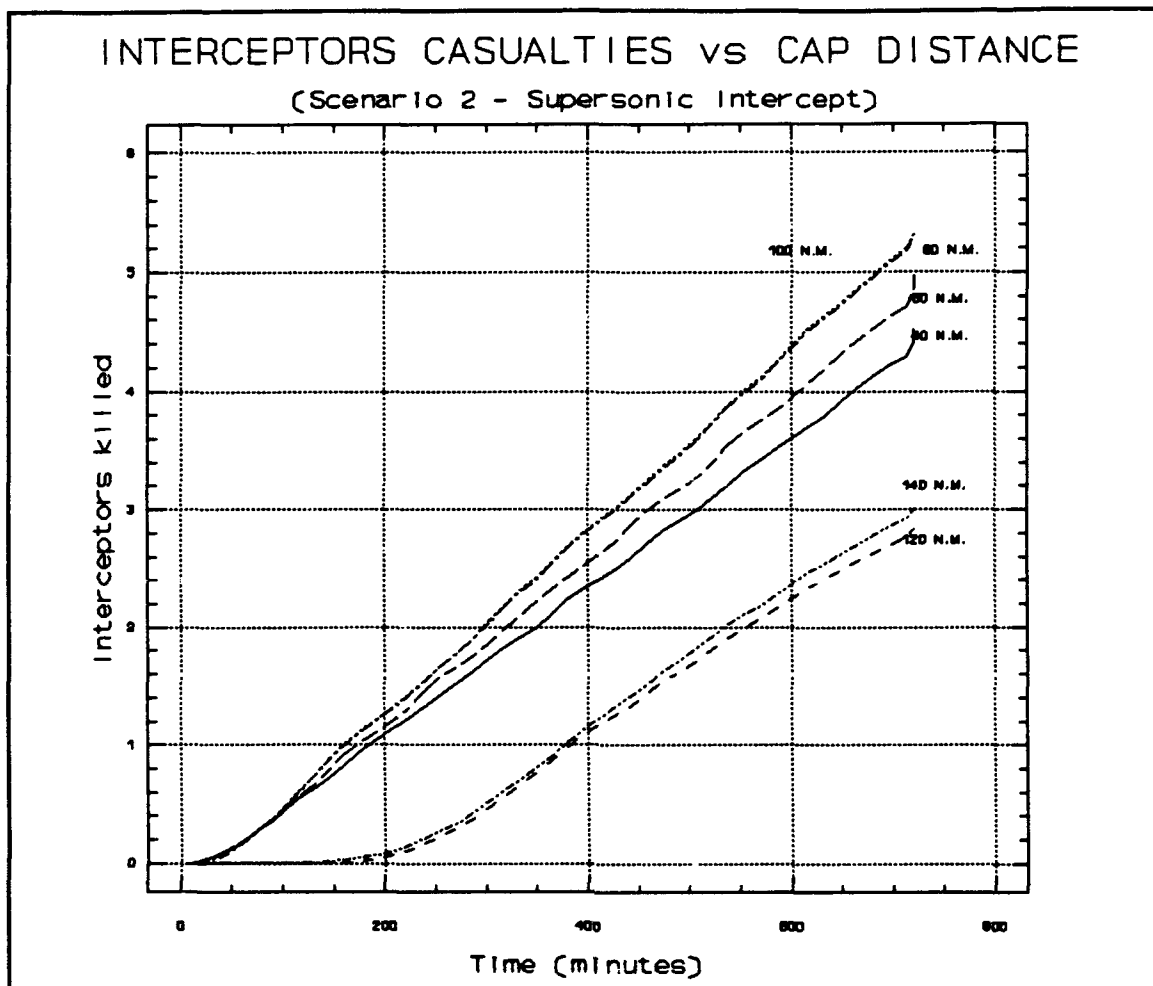


Figure 15. Scenario 2 (supersonic) MOE2: Interceptors Casualties

intercept phase. Because the fuel for combat shall not be used in any phase of the mission other than the combat engagement itself, all the fuel demand excess is supplied by the fuel for the loitering phase on CAP. In some situations this causes the fuel for loitering to be consumed by the fuel excess demand generated by the scenario specificities, depleting it to the zero level; and when this happens there will be no interceptor on station. At the model level this fact is represented by the conditions $I_{CAP} = 0$, and $N_C(t) < \bar{z}$. When the model detects these conditions

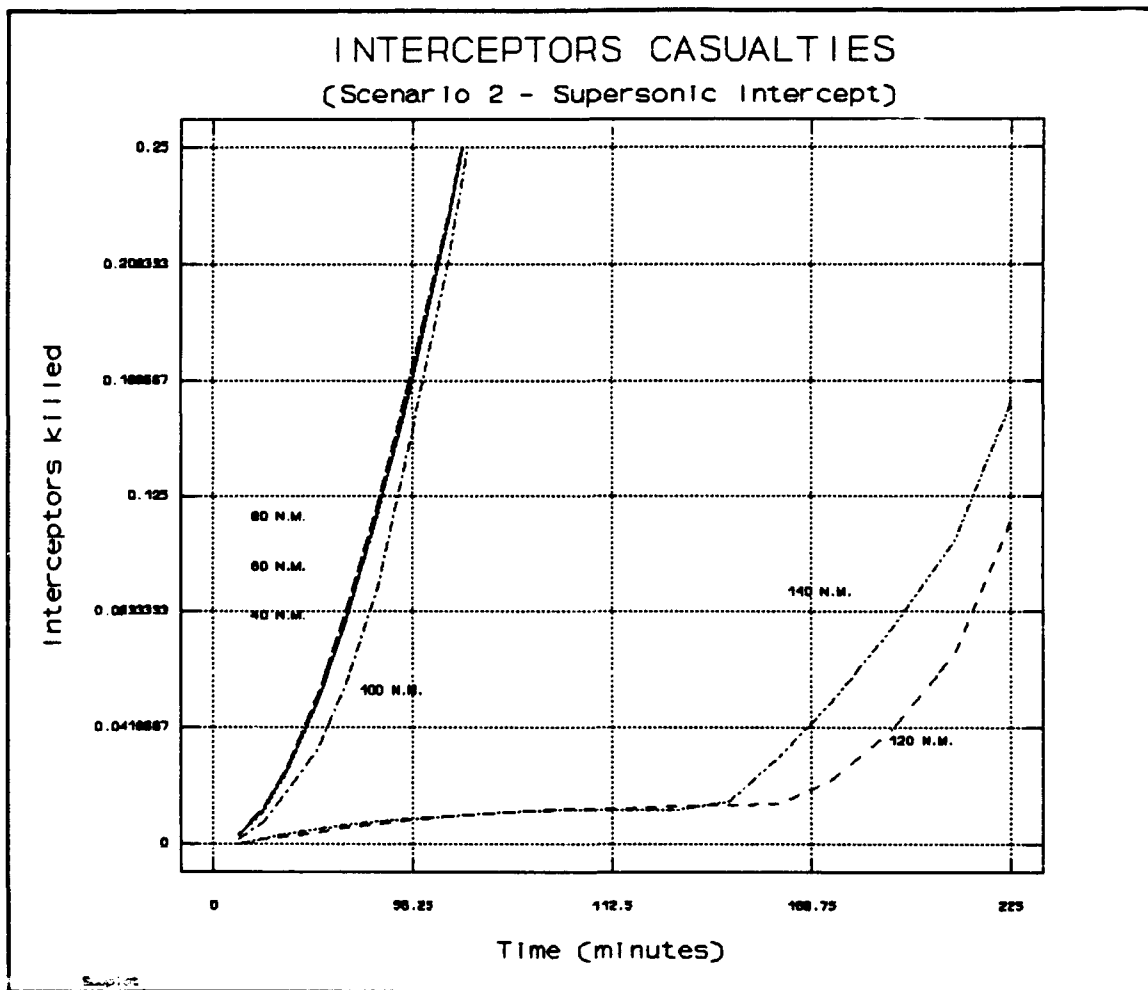


Figure 16 Subplot from Figure 15

another interceptor is launched from the air base to go to CAP station, but this interceptor will not loiter on station either, so there will be a constant flow of transient interceptors from air base to CAP station back and forth. Under these conditions all the engagements that take place are those involving the transient interceptors which, due to the geometry of the problem, yield a lower pace attrition rate than that for the engagement from CAP station. Hence, according to Figures 12 and 15, stationing the CAP at further distances and using supersonic air speed setting

for the intercept will render fewer casualties on both sides of the battle. This explains the lower kill rates in Figures 12 and 15 for the far out CAP options.

In Figure 17 below we have the results regarding MOE3 for scenario 2 with subsonic intercept.

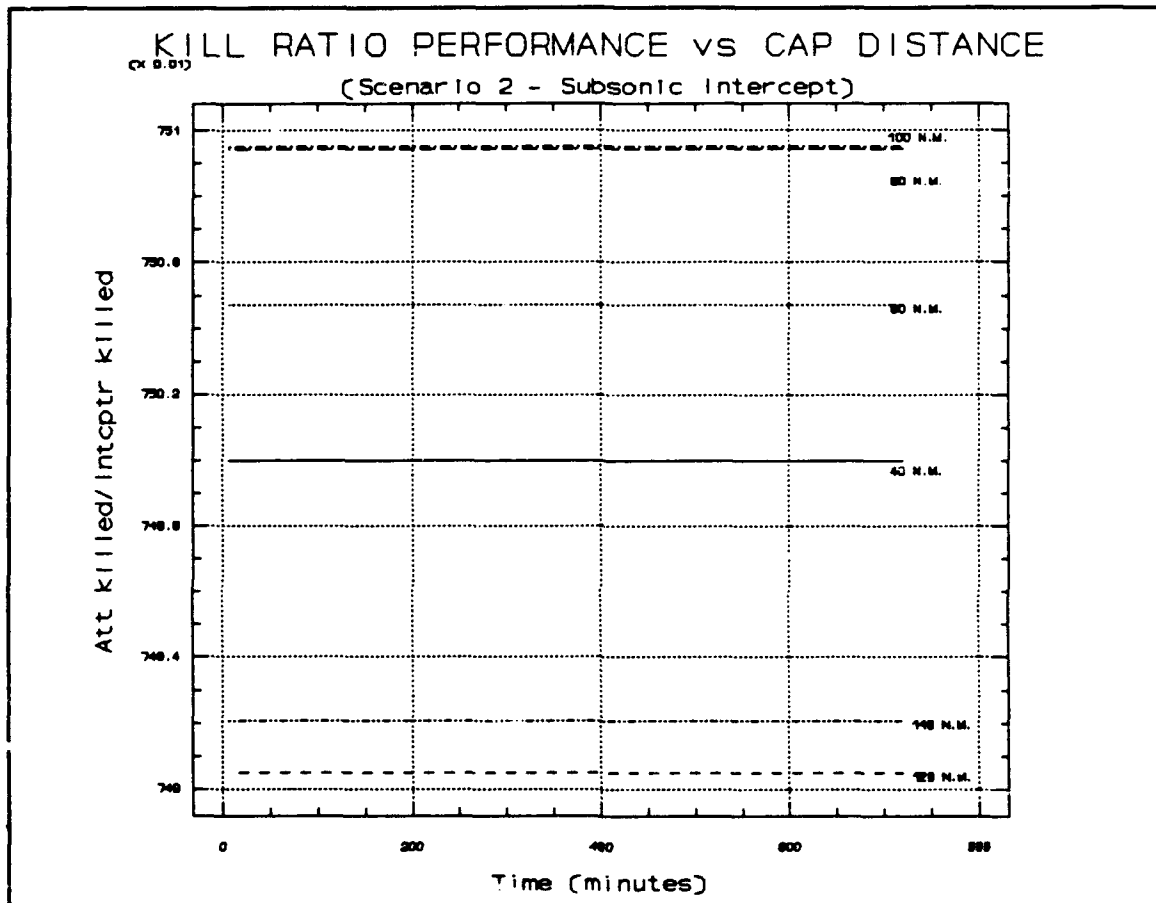


Figure 17. Scenario 2 (subsonic) MOE3: Kill Ratio

Considering the nature of MOE3, the option with best performance is to place the CAP either at the 60 N.M. or at 100 N.M. ranges. But if we observe the magnitude of the values on the y-axis scale, it is clear that, as far as a decision is

concerned, the differences in performance among the options has no physical meaning.

Figure 18 shows the results for MOE3 for the supersonic intercept in scenario 2. Figure 19 magnifies Figure 18.

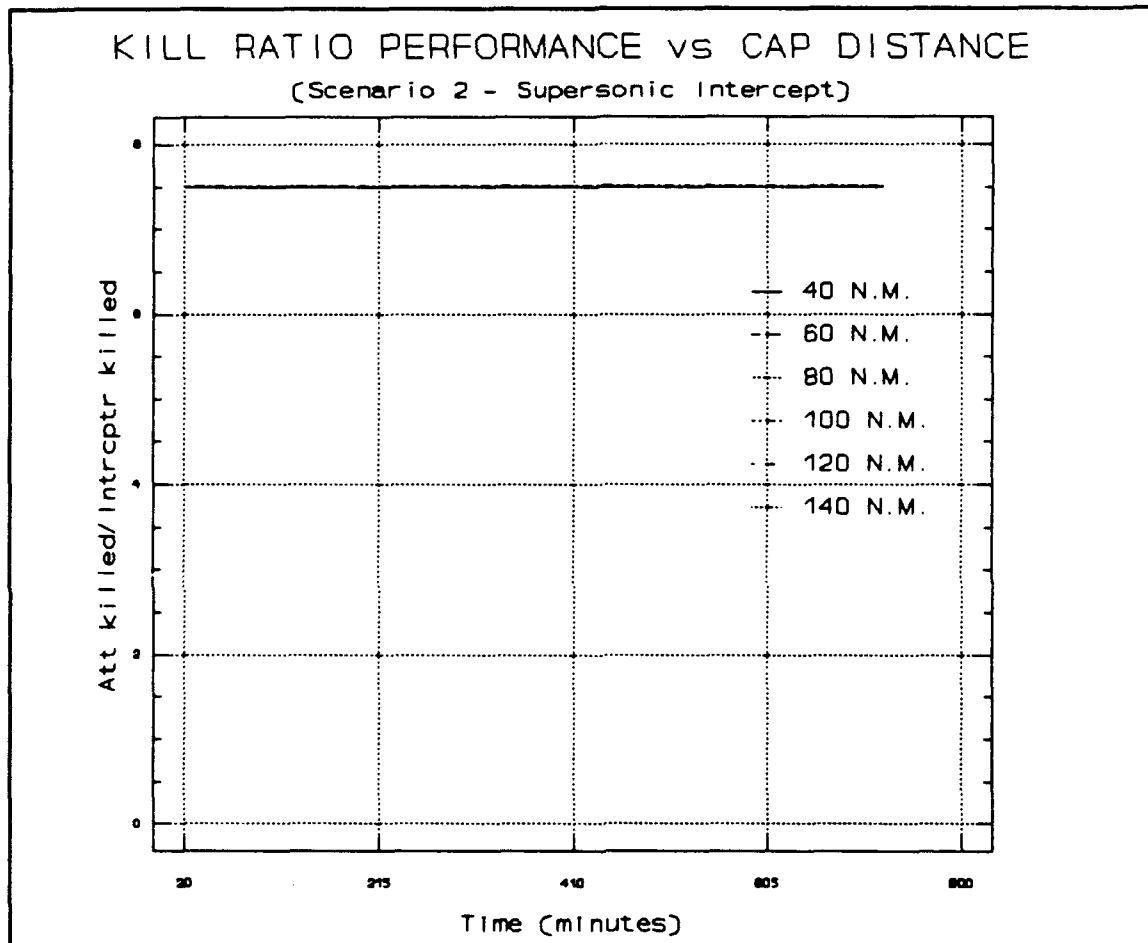


Figure 18. Scenario 2 (supersonic), MOE3: Kill Ratio

It can be noticed that, besides the numerical differences shown in Figure 19, in this scenario also the air defense planner can not decide among the options based only on this MOE. target area longer then 80 N.M., during the initial phase of the battle the attackers can pass through the area of CAP responsibility at no charge.

Regarding the other options, it is seen that besides the differences shown in MOE1 and in MOE2, the net benefits of each of these options are equivalent.

Figure 19 shows the results for MOE4 from scenario 2 with the subsonic intercept.

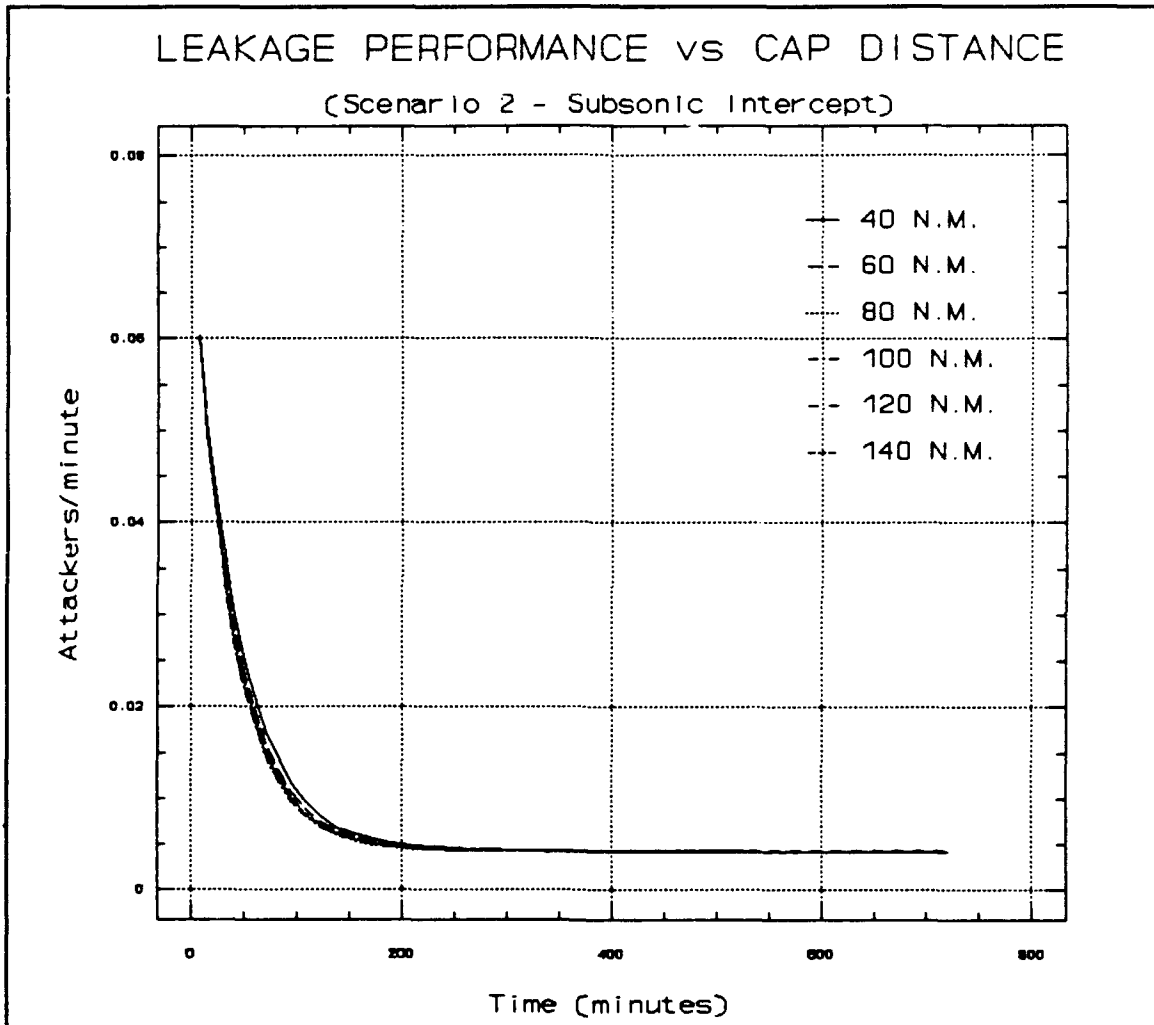


Figure 19. Scenario 2 (subsonic), MOE4: CAP Leakage

The results regarding MOE4 look similar for all options, according to Figure 19. There is a significant drop in the leakage level in the initial phase of the CAP activity and a leveling out from the beginning of the third hour of battle. To make

it possible to distinguish among the options we will magnify that portion of Figure 19 where the initial decrease occurs. In Figure 20 we have a subplot of Figure 19 containing such a zooming.

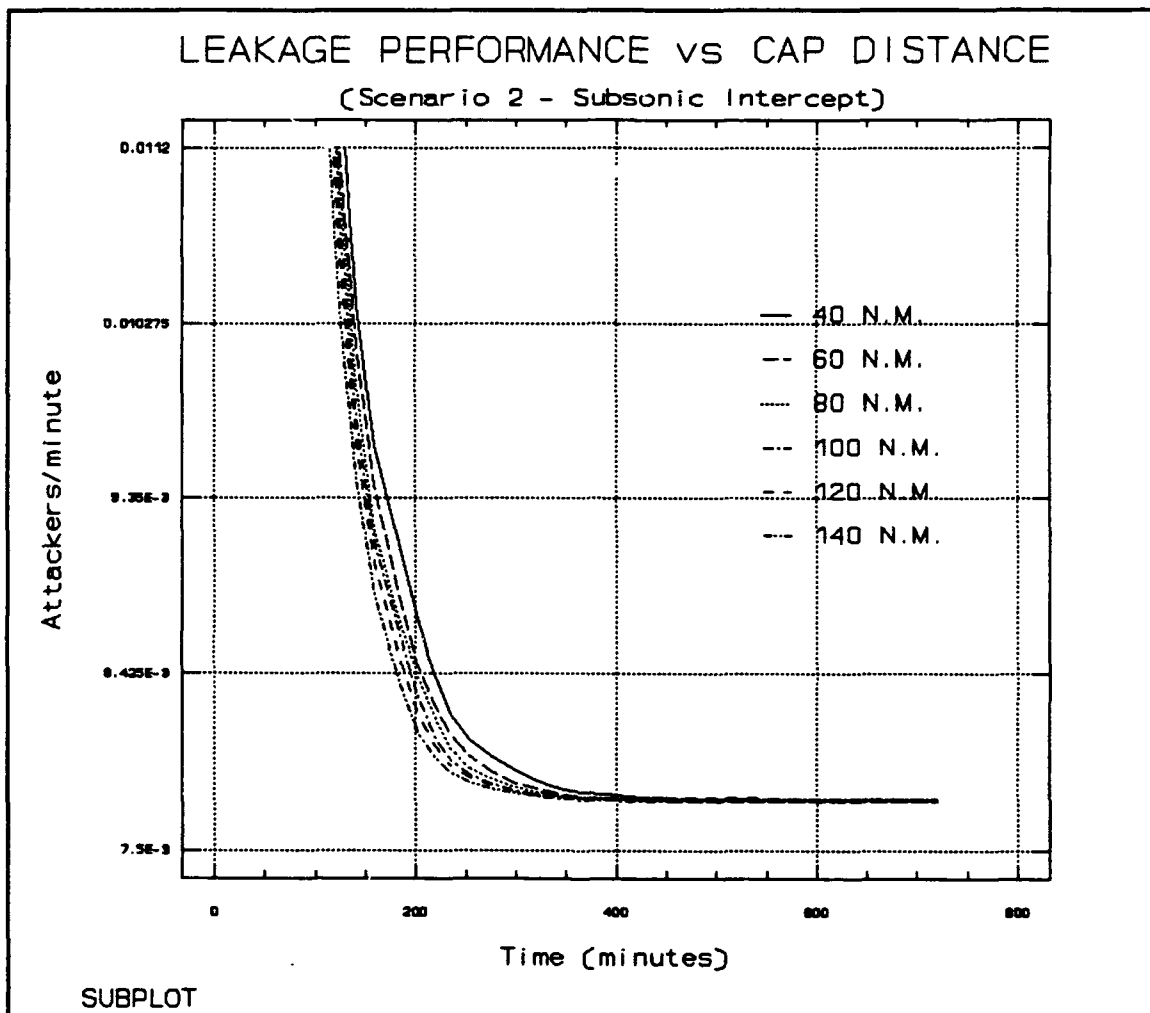


Figure 20. Subplot from Figure 19

The analysis of Figure 20 shows that besides the similarities, with the option for stationing the CAP at 40 N.M., between the third and the forth hour of battle on, the attackers will be surviving the CAP interceptors at a rate of 0.008994 aircraft per minute, what is equivalent to one attacker being engaged by the ani-air weapons

system each 111 minutes. If the CAP is stationed at 140 instead, the leakage rate by the same time will be around 0.008062 attackers per minute, i.e., one attacker being engaged by the anti-air weapons system each 124 minutes. By the seventh hour of battle, disregarding the option adopted, the leakage rate level out at a rate around 0.00781, meaning one attacker each 128 minutes. If we consider that the attackers are arriving at a constant rate of one attacker each 15 minutes, we can see that, in this scenario the CAP interceptors represent a great relief for the ground-based weapons.

MOE4 for the same scenario with supersonic intercept is presented in Figure 21 below. In this case the options for the further CAP stationing show the worst performances. Observe that in the initial phase of the battle, with the CAP placed at the further ranges, the leakage rate almost the same as the attackers' arrival rate, what is consistent with the previous analysis. In Figure 22 we have the zooming of of the plot in Figure 21 focusing on the closer CAP options. If the CAP is stationed at 100 N.M., by the one hundredth minute of battle the leakage rate will be around 0.01271 aircraft per minute, meaning one ground weapons engagement at each 78 minutes. If the CAP is at 40 N.M., by this time this leakage rate will be around 0.01375 aircraft per minute, meaning one ground weapon engagement at each 72 minutes; with this option the leakage rate stabilizes by the fourth hour of battle at value 0.00794, yielding an engagement rate for the anti-air weapons of one aircraft each 126 minutes.

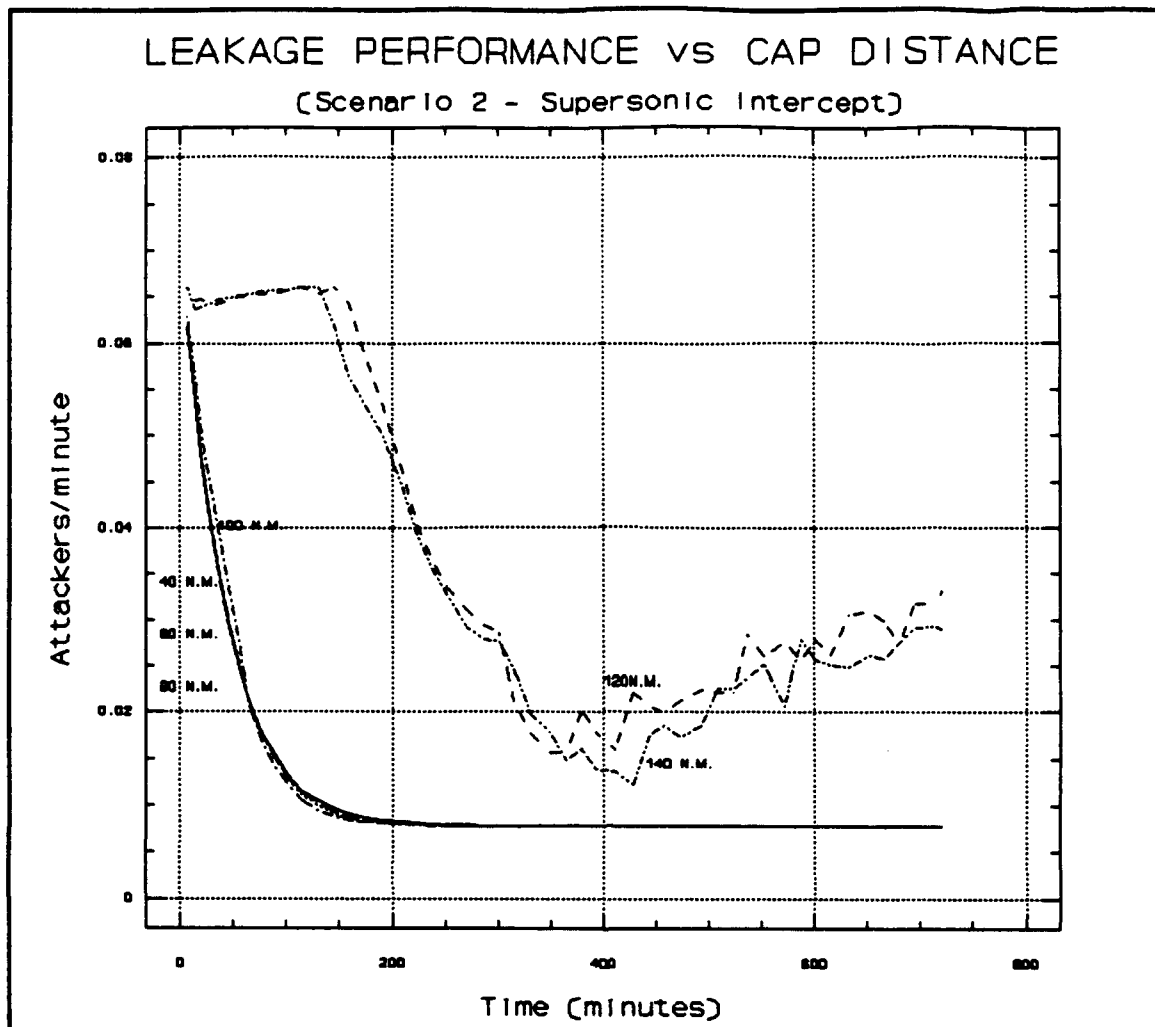


Figure 21. Scenario 2 (supersonic), MOE4: CAP Leakage

Regarding the further away options, Figure 21 shows that they do not stabilize the leakage rate at any specific value. There is the initial drop, reaching the lowest level after the sixth hour of battle, after what we can see an increase trend. At no point in time do they perform better than the closer CAP options.

The analysis of these plots can be used as a decision aid tool by the air defense staff.

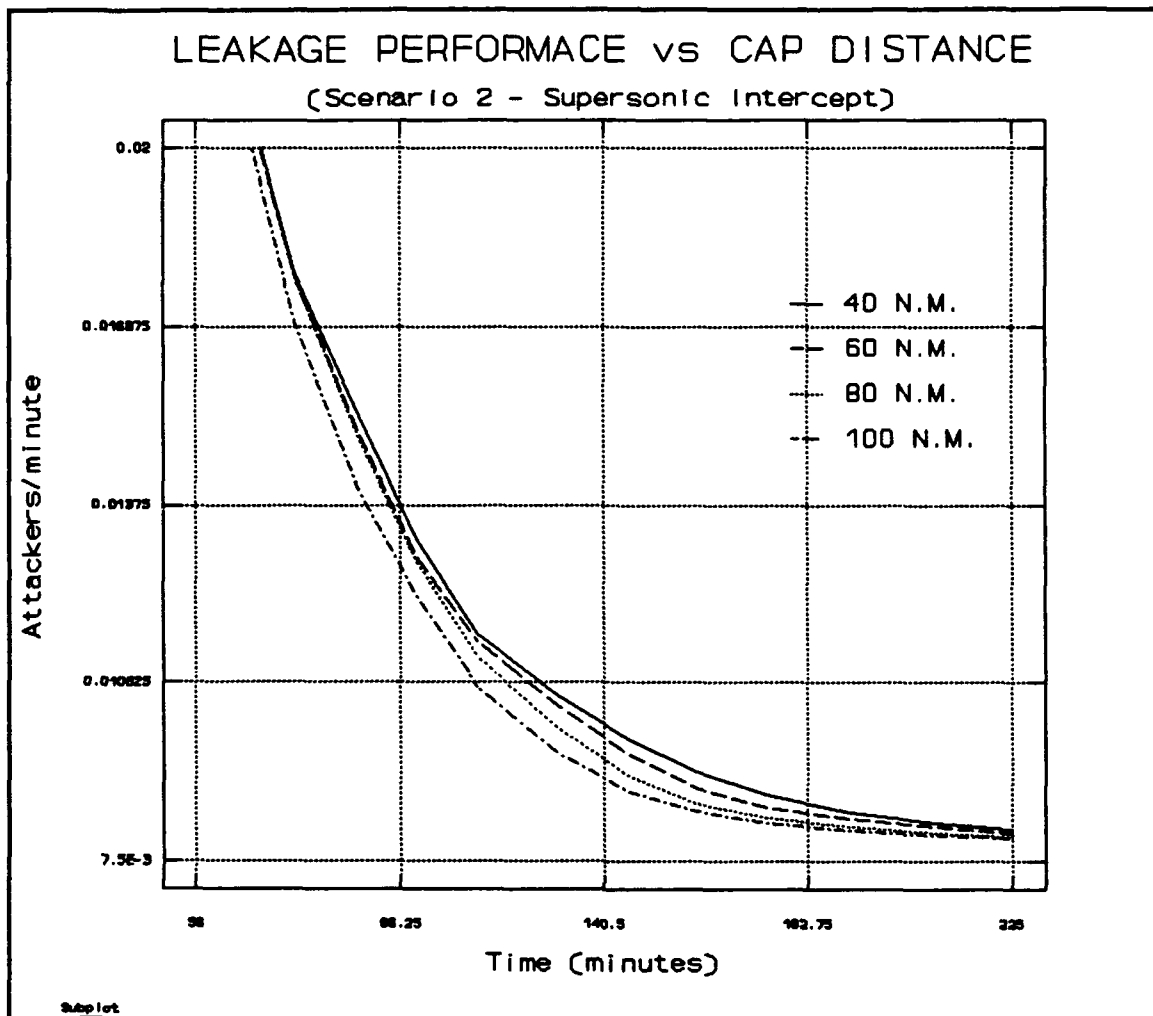


Figure 22. Subplot of Figure 19 for the short range CAP.

Suppose that in scenario 1 the defense is facing aircraft shortage. Then the best option would be to place the CAP station at 30 N.M. because Figure 8 shows that this is the CAP range yielding the least casualty level for the CAP interceptors. If the intention is to inflict as many casualties to the attackers as possible, then Figure 7 shows that the best tactic would be to station the CAP at the 50 N.M. range because this option renders the highest attackers's casualties level in the time. If the defense knows about the attacking air force aircraft inventory levels, then the plot

of MOE3 is useful because it gives an idea of the casualties relationship between both sides, allowing some kind of forecasting capability to the air defense planners. If it is the case that the anti-air weapons system is under some kind of constraint, then Figure 10 picturing MOE4 can show the option rendering the least CAP leakage rate.

The above considerations about the uses of the model's results apply in the very same way to scenario 2, or to any kind of scenario where the reality of interest is represented in the model's parameters. It is also a useful tool to evaluate the performance of two distinct tactic options in the same scenario. As it becomes clear from these numerical examples, the use of supersonic intercepts in scenario 2 is not the most effective tactic for the defense⁶.

⁶ With this regard it must be pointed out that in this numerical examples, "subsonic" means the interceptor flying at Mach Number 0.92, and "supersonic" means that the interceptor is flying at Mach Number 1.2.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

This study is an investigation of the Combat Air Patrol stationing problem in the tactical scenario. The geometry of a hypothetical scenario was analyzed and equation (3.7) was derived to determine the minimum early warning radar detection range required for intercept feasibility in a generic scenario. This equation considers any air speed relationship between the attacker and interceptor aircraft, and assumes the air base located either inside or outside of the target area. Using this expression the necessity of using the interceptors in CAP in a given situation can be identified.

Considering an interceptor in Combat Air Patrol in a given position, equation (3.8) was derived to express the intercept range as measured from the target area position as a function of: the early warning detection range; the system's identification time; the distance of the CAP station from the target position; the interceptor/attacker air speed relationship; and the attacker's flight path angle, as measured with respect to the CAP station position.

The minimum number of interceptors required to activate one CAP station was addressed. For this end equation (3.13) was derived and expresses this number as a function of the time variables involved in the dynamic process that a CAP station management represents. These time variables were investigated and the maximum time for combat (t_{cmb}) and the maximum time on CAP station (t_m) were identified

as the key variables for analysis. In addressing these two time variables, the fuel consumption problem had to be considered.

The analysis regarding t_{cmb} dealt with the aircraft maneuverability and turning performance comparisons based on the energy-state approximation theory; a heuristic method for computing t_{cmb} which relates this time length to an expected interceptor's performance during the air-to-air combat engagement was postulated. The time length determined this way is translated into an amount of fuel and into a distance measure according to equation (3.15), and these two quantities are used in the air defense planning process. In the analysis of t_m the interceptor/attacker air speed relationship was identified as a factor that ultimately affects the number of aircraft required to activate one CAP station. A third party investigation result was presented showing that high settings of intercept air speed is of little value for the intercept distance.

In order to capture the complexities present in the dynamic process embedded in a Combat Air Patrol management, a deterministic model representing such a process was developed. The state variables of this model were defined so as to portray both the logistic and the operational aspects and constraints of such an activity. These types of realities are represented in the model by means of the system's parameters controlling the transition flow from one state to another. A method for computing the attrition rate was derived based on Bonder and Farrell's methodology. Numerical examples considering two different scenarios and three distinct situations were presented and the results analyzed. The utility of such model

for the CAP problem analysis in an air defense planning process was addressed. The model is preliminary and will require further work to become an useful planning tool.

B. RECOMMENDATIONS FOR FURTHER RESEARCH

The interceptor's fuel consumption characteristics were identified as one dominant factor in the analysis of the CAP stationing problem. One way of improving a CAP interceptor fuel capability is to resort to air refueling. Such a possibility was not considered in the present study and neither was it represented in the deterministic macro model. The impact of the air refueling option in the overall CAP interceptor performance is worth additional study.

A key assumption to compute the attrition rate coefficients for the deterministic model is that the *interceptor aircraft* uses all its ammunition in each engagement. This assumption is reasonable depending on the air-air weapons load capabilities of the interceptor aircraft represented in the model and led to a simplified method of attrition rate computation. If such assumption is relaxed a greater variety of aircraft could be represented in the model. However such relaxation would require that the method used here to compute the attrition rates be revised; this is recommended for further analysis.

The deterministic macro model for CAP management was applied in numerical examples for which the scenarios considered the air base supporting one single CAP station activated, and an homogeneous type of threat. The issue of multi-threat

sectors covered by CAP interceptors supported by a single air base is worth addressing in order to assess the effects of resource allocation and priority policies on the overall efficiency of the CAP interceptors in different sectors of threat.

C. CONCLUSIONS

The present investigation has shown that to decide on a better way to station interceptor aircraft in Combat Air Patrol in a tactical scenario defended by an Integrated Air Defense System, requires an analysis that considers many different factors. Some of these factors are scenario-dependent, others are functions of the characteristics of the resources available. A judicious decision process must take into account the interactions between these different elements of the problem and identify those most significant for the measure of effectiveness selected.

The present study identified the interceptor's fuel usage characteristics as a dominant factor in all aspects of the problem. All time variables identified as being to some extent germane to the problem are subject to fuel constraints. The maximum length of time the interceptor can stay on CAP station, and the time available for air-to-air combat engagement, are the time variables with the greatest relevance in the CAP interceptor performance. The former because it impacts the demand of aircraft for the CAP mission, and the latter because it affects the interceptor's lethality.

There is not a unique solution for a problem with so many facets. A deterministic macro model using the mean values of the variables involved was developed in an attempt to shed light on the interactions among the logistics and the

operational variables present in the phenomenon. It was shown that in a tactical environment, the use of high values for the intercept air speed decreases the CAP interceptor overall contribution for the air defense mission. In spite of approximate nature of the results, the model developed may be adapted to be useful for many practical purposes, either as tool for planning analysis or as an aid in a decision process.

APPENDIX

In what follows there is a description of some additional air-to-air models found in the review of the unclassified literature.

Riecks [Ref. 14] addressed the interactions between airborne interceptors and penetrators in two distinct models. The main purpose of these models is to evaluate a bomber's defensive missiles as penetration aids to bombers carrying cruise missiles. The scenario for both models utilizes the corridor penetration concept, in which the bombers enter a corridor covered by the Forward Air Defense (FAD) comprising a single Airborne Warning and Control System (AWACS) controlling airborne interceptors on CAP. One model is a simulation using the Q-GERT simulation language, in which the battle is represented as a queue system where bombers and interceptors wait on queue to be paired by the AWACS who acts as server. The other model is a stochastic analytic approach recursively estimating a separate survival probability for each successive bomber to enter the corridor. The analytical model derives expressions to compute quantities such as:

$P_s(j)$, the probability that a bomber survives, given j AI engagements; [Ref. 14:pp. 66-73]

$E_{pic}(i)$, the expected number of penetrators (bombers and Air Launched Cruise Missiles - ALCM) in AWACS coverage during the i^{th} bomber's attempt to penetrate the FAD; [Ref. 14:pp. 73-80]

$N_{ai}(i)$, the expected number of fighters alive during the i^{th} bomber's time in coverage;[Ref. 14:pp. 81-87]

$P_{ai}(i,k)$, the probability that k is the maximum number of intercepts that can be attempted against the i^{th} bomber;[Ref. 14:pp. 88-89]

E_{mi} , the estimated total number of ALCM's launched;[Ref. 14:pp. 90-91] and

E_{ms} , the expected number of cruise missiles surviving.[Ref. 14:p. 92]

In both models the AI's are considered to be on CAP station at the same positions as the AWACS, and the air base is located at a given distance from the CAP. The model captures many aspects of a many versus many air-to-air engagement, including defense saturation. Nevertheless, the effect of air base-CAP station distance is not analyzed.

Clements [Ref. 9] analyzes another aspect of the air-to-air battle, namely the maximum number of interceptors that can be simultaneously controlled in a theater tactical air defense scenario. According to the author, this number was needed by the Air Force Center for Studies and Analysis as an input variable for their new theater air combat simulation, TAC ALLOCATOR. To this end, the author used data collected in a survey amongst USAF's air weapons controllers. The analysis of the research questionnaire indicated that the average air weapons controller is capable of simultaneously controlling either 3.41 or 4.71 flights of interceptors depending on the interceptor on-board radar capability [Ref. 9:p. 5-2]. Using these results a computer simulation was designed in which the scenario represented was a Central European theater. In this scenario many CAP stations were considered simultaneously, but the CAP locations were not specifically addressed in the study.

A 2-factor experimental design was run and the conclusion was that if no radar/communications jamming is considered, if the penetrators do not have lethal self-defense capability, and if the enemy raids are uniformly distributed throughout the theater, then the multiplication of the control capability of the average air weapons controller by the number of controllers available accurately predicts the maximum control capability of the system [Ref. 9:p. 6-2]

Next we have a general description of the main characteristics of some air-to-air models found in a research made by Grant [Ref. 3].

Advanced Penetration Model (APM) Is a model developed by the Boeing Corporation for Headquarters USAF in the early 70's. It models all aspects of the bomber mission: takeoff, base fly out, refueling, forward air defense, SAM zones, terminal and point defenses, weapons delivery at target and recovery at a friendly air base. The model features two major segments: the Mission Planner which is used to define the overall scenario and generates the individual flight plans, and the Air Battle Simulator which implements, in time sequence, the events that have been specified in the mission planner and inserts others as required by deterministic or probabilistic event assignment. According to the author, APM produces a great amount of information in the form of a time history of each event, allowing the user to examine defense saturation, command and control limitations and weapons assignment policies. On the other hand, this detail is also a model weakness: opportunity for multiple replications is limited because of computer time costs.[Ref. 3:pp. 6-7]

Simulation of Penetrators Encountering Extensive Defenses (SPEED) This model was developed by Calspan Corporation in order to be a fast running, smaller scale version of the APM. It addresses only the part of the bomber mission dealing with the forward air defense. The offensive elements include manned bombers, guided missiles, drones, decoys and short range missiles and bombs. The defensive elements include a C² netting of early warning (EW), ground control intercept (GCI) radars , and a zone operations center (ZOC). The ZOC pairs penetrators with interceptors. AWACS aircraft orbits between two points at an altitude defined by the user. There is a CAP associated with the AWACS from which the AWACS assigns interceptors to the penetrators it detects. The model assesses the efficiency of the interceptors by the percentage of time bombers are engaged by interceptors and the percentage of time bombers are not engaged owing to time delays in the defense C² system, ECM effects, or saturation of the communication channels for the interceptors. According to the author, a weakness of the SPEED is the number and detail of the parameters which the user must provide.[Ref. 3:pp. 7-8]

FISHER or STRAT DEFENDER This model was developed by William Fisher of the North American Aerospace Defense Command (NORAD). Later modifications introduced by AF/SD (Strategic Aerospace Defense Division) HQ USAF led to renaming the model as STRAT DEFENDER. It is a simulation model addressing forward air defense involving unarmed penetrators against fighters and SAM defenses. Aspects such as radar cross-section, speed, altitude and other

features of the penetrators, detection and armament capabilities of the interceptors, and detection range of the AWACS and GCI are taken into account to decide if a penetrator is liable to interception. Interceptors are vectored from CAP and from air bases to the penetrator, and detection, conversion and kill actions are determined stochastically. The model also includes aspects such as air base location and their effects on interceptor fuel and armament requirements.[Ref. 3:pp. 8-9]

STRAT PATROLLER This model was developed by General Research Corporation for AF/SD, HQ USAF, in 1979. It is an event-based simulation of interceptors flying air surveillance, and is intended to provide an autonomous interceptor modeling facility to the FISHER model. The major focus of the model is the detection function of a surveillance barrier. According to the author, by the time her research was being done, STRAT PATROLLER was still in development.[Ref. 3:p. 9-10]

PENEX This is an analytic model which, using principles of probability theory, calculates the expected number of bombers surviving in a many-on-many air battle with manned interceptors. The scenario comprises a corridor in which the air battle takes place. The penetrators have identical performance, may carry decoys to be released along the way, and neither bombers or decoys will fire at the interceptors. The interceptors are all identical to one another and they can not distinguish between bombers and decoys. Command and control is modeled according to two different philosophies: raid control and close control. One weakness of the model,

according to the author, is that it does not model interceptor availability as a function of time or the number of control channels available.[Ref. 3:pp. 10-11]

COLLIDE This is an analytical penetration model developed by Decision Science Applications for AF/SD HQ USAF, in 1972. It models the air-to-air combat according four submodels: a detection model; a conversion model; a command and control model; and an ECM model. It focuses on finding the probabilities of detection, conversion and kill based on a one-to-one encounter. To compute each one of these probabilities it takes into account factors as altitude differences, type of sensor, relative velocities, angularly dependent probability of kill and ECM effects. The considerations of all these details in computing the probabilities makes the strength of this model. On the other hand, according to the author, the fact that it models a one-on-one encounter makes it difficult to expand the model into a many-on-many scenario as in the FAD models.[Ref. 3:pp. 11-12]

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